Textbook problem **6.5.A**:

The impulse of a force action on a body changes its momentum by

$$\Delta\mathbf{P} \equiv \mathbf{P}_{\text{after}} - \mathbf{P}_{\text{before}} = \mathbf{I}. \tag{1}$$

In all three examples, the body stops moving, hence $\mathbf{P}_{\text{after}} = \mathbf{0}$ and therefore

$$\mathbf{I} = -\mathbf{P}_{\text{before}} = -M\mathbf{v} \tag{2}$$

where $\mathbf{v}$ is the body's velocity before the collision. As far as the impulse's magnitude is concerned,

$$I = Mv. \tag{3}$$

In particular:

(a) for the hockey puck, $I = 0.5\ \text{kg} \times 35\ \text{m/s} = 17.5\ \text{kg}\cdot\text{m/s}$;

(b) for the tennis ball, $I = 0.2\ \text{kg} \times 15\ \text{m/s} = 3\ \text{kg}\cdot\text{m/s}$;

(c) for the tank, $I = 12\cdot 10^3\ \text{kg} \times 4\ \text{m/s} = 48\cdot 10^3\ \text{kg}\cdot\text{m/s}$.

Textbook problem **6.5.B**:

The impulse of a force is defined as the average value of the force times its duration,

$$\mathbf{I} = \overline{\mathbf{F}} \times \Delta t. \tag{4}$$

Hence, given the impulse and the duration of the force, the average force is

$$\overline{\mathbf{F}} = \frac{\mathbf{I}}{\Delta t}. \tag{5}$$

Focusing on the magnitude of the average force and ignoring the direction, we drop the vector notations and have $\overline{F} = I/\Delta t$. For the examples in question:

(a) for the hockey puck, $\overline{F} = (17.5\ \text{kg}\cdot\text{m/s})/(1\ \text{s}) = 17.5\ \text{N}$;

(b) for the tennis ball, $\overline{F} = (3\ \text{kg}\cdot\text{m/s})/(0.1\ \text{s}) = 30\ \text{N}$;

(c) for the tank, $\overline{F} = (4.8\cdot 10^3\ \text{kg}\cdot\text{m/s})/(2\ \text{s}) = 24\cdot 10^3\ \text{N}$.

Note units: $1\ \text{kg}\cdot\text{m/s} = 1\ \text{N}\cdot\text{s}$, hence $(1\ \text{kg}\cdot\text{m/s})/(1\ \text{s}) = 1\ \text{N}$.

Textbook problem **6.5.C**:

There is no single rule for all types of damage, but most of the time the damage follows from the force itself rather than its impulse. Indeed, a big force acting over a short time makes a lot more damage than a small force acting over a long times, even if the net impulse $F \times \Delta t$ is the same in both cases.

The area over which the force acts also affects the damage: The force concentrated over a small area will be a lot more damaging than the same force spread over a large area. On the other hand, concentrating the force on a particular body part also concentrates the damage to that part, so the net damage depends on how important is that part for the whole body.

Textbook problem **6.8**:

Throw the oranges in the forward direction as fast as you can — if you get a baseball bat or something else which can give them more speed, use it. The point of this exercise is the recoil, which will slow down the boat; if you throw enough oranges fast enough, the boat will stop. Note that without friction, nothing else will work: the net momentum of the boat and the oranges is conserved, so if you don't throw them (or something else) overboard, the boat will continue moving at constant velocity

$$\mathbf{v} = \frac{\mathbf{P}^{\text{net}}}{M_{\text{total}}} \tag{6}$$

until it falls into the crevice. Also, you cannot just throw the oranges overboard, you must give them forward velocity faster than the boat's: this way, the oranges gain forward momentum while the rest of the boat loses momentum and slows down.

Let v^0 be the initial velocity of the boat (with all the oranges in it), v'_{oranges} the velocity of the oranges thrown overboard, and the v'_{boat} the velocity of the boat after the oranges are gone; all velocities are relative to the ice. Computing the net momentum before and after throwing the oranges, we have

$$\begin{aligned} P_{\text{before}} &= \bigl(M_{\text{total}} = M_{\text{boat}} + M_{\text{oranges}}\bigr) \times v^0, \\ P_{\text{after}} &= M_{\text{boat}} \times v'_{\text{boat}} \;+\; M_{\text{oranges}} \times v'_{\text{oranges}}. \end{aligned} \tag{7}$$

By momentum conservation, $P_{\rm after} = P_{\rm before}$ and therefore

$$M_{\rm boat} \times v'_{\rm boat} \; + \; M_{\rm oranges} \times v'_{\rm oranges} \; = \; \bigl(M_{\rm boat} + M_{\rm oranges}\bigr) \times v^0, \tag{8}$$

or equivalently

$$M_{\rm boat} \times \bigl(v^0 \; - \; v'_{\rm boat}\bigr) \; = \; M_{\rm oranges} \times \bigl(v'_{\rm oranges} \; - \; v^0\bigr)\,. \tag{9}$$

Thus, if you throw the oranges forward so that $v'_{\rm oranges} > v^0$, then $v'_{\rm boat} < v^0$ and the boat slows down.

To completely stop the boat we need $v'_{\rm oranges}$ so high that $v'_{\rm boat}$ becomes zero. Plugging $v'_{\rm boat} = 0$ into eq. (9) and solving for the $v'_{\rm oranges}$, we find

$$v'_{\rm oranges} \; = \; v \times \frac{M_{\rm boat} + M_{\rm oranges}}{M_{\rm oranges}}. \tag{10}$$

If you cannot throw the oranges this fast, you cannot stop the boat.

Non-textbook problem #1:

Again, we use momentum conservation, $\mathbf{P}^{\rm after}_{\rm net} = \mathbf{P}^{\rm before}_{\rm net}$: the net momentum before and after the collision is the same. Before the collision we have

$$\mathbf{P}^{\rm before}_{\rm net} \; = \; m_{\rm car}\mathbf{v}_{\rm car} \; + \; m_{\rm cow}\mathbf{v}_{\rm cow} \; = \; m_{\rm car}\mathbf{v}_{\rm car} \tag{11}$$

because cow's velocity before the collision is zero (she is standing on the road). After the collision, the car and the cow move at the same velocity $\mathbf{v}'$ (the cow on the car's hood, she does not have a choice), hence

$$\mathbf{P}^{\rm after}_{\rm net} \; = \; m_{\rm car}\mathbf{v}'_{\rm car} \; + \; m_{\rm cow}\mathbf{v}'_{\rm cow} \; = \; (m_{\rm car} \; + \; m_{\rm cow})\,\mathbf{v}'. \tag{12}$$

Therefore, by momentum conservation

$$(m_{\rm car} \; + \; m_{\rm cow})\,\mathbf{v}' \; = \; m_{\rm car}\mathbf{v}_{\rm car}\,, \tag{13}$$

and consequently

$$\mathbf{v}' \;=\; \frac{m_{\text{car}}}{m_{\text{car}} \;+\; m_{\text{cow}}} \times \mathbf{v}_{\text{car}}\,. \tag{14}$$

Numerically, speed after the collision is

$$v' \;=\; \frac{2000\,\text{lb}}{2000\,\text{lb} \;+\; 1000\,\text{lb}} \times 60\text{MPH} \;=\; \frac{2}{3} \times 60\text{MPH} \;=\; 40\text{MPH}. \tag{15}$$

<u>Non-textbook problem #2</u>:

Elastic collision of two bodies — such as steel balls — satisfies two conditions: (a) same relative speed before and after the collision,

$$\Big|\mathbf{v}_1 - \mathbf{v}_2\Big| \;=\; \Big|\mathbf{v}'_1 - \mathbf{v}'_2\Big|, \tag{16}$$

and (b) momentum conservation

$$m_1\mathbf{v}_1 \;+\; m_2\mathbf{v}_2 \;=\; m_1\mathbf{v}'_1 \;+\; m_2\mathbf{v}'_2\,. \tag{17}$$

For the collision in question we know both balls' velocities before and after the collision but we don't know the masses. Consequently, the elasticity condition (a) is not useful: we may check that it is satisfied — it is — but it does not tell us anything about the masses. Instead, we use the momentum conservation rule (b) — it applies to any collision, elastic or inelastic.

Let us rearrange eq. (17) by moving all term related to the first ball to the left hand side and terms related to the second ball to the right hand side:

$$m_1 \times \Big(\mathbf{v}_1 \;-\; \mathbf{v}'_1\Big) \;=\; m_2 \times \Big(\mathbf{v}'_2 \;-\; \mathbf{v}_2\Big). \tag{18}$$

This is a one-dimensional problem so we may drop the vector notation, but we must continue to keep track of velocities' signs which indicate the direction of motion. Thus,

$$m_1 \times \Big((+10\,\text{m/s}) \;-\; (-5\,\text{m/s}) \;=\; +15\,\text{m/s}\Big) \;=\; m_2 \times \Big((+5\,\text{m/s}) \;-\; (0\,\text{m/s}) \;=\; +5\,\text{m/s}\Big), \tag{19}$$

and consequently

$$\frac{m_1}{m_2} \;=\; \frac{5\,\text{m/s}}{15\,\text{m/s}} \;=\; \frac{1}{3}\,: \tag{20}$$

the first ball is 3 times lighter than the second ball.