Non-textbook problem #1:

(a) The period of rotation is the inverse of frequency,

$$T \;=\; \frac{1}{f}\,. \tag{1}$$

Before we evaluate this formula, let us convert the frequency from RPM=rev/min to rev/s: $f = 10080\text{rev/min} \times (1\text{ min}/60\text{ s}) \;=\; \frac{10080}{60}$ rev/s $=$ 168 rev s. Consequently,

$$T \;=\; \frac{1}{f} \;=\; \frac{1\text{ rev}}{168\text{ rev/s}} \;=\; \frac{1\text{ s}}{168} \;\approx\; 6\text{ ms}. \tag{2}$$

(b) Since 1 complete revolution is a rotation by 2π radians, the angular velocity ω in rad/s is 2π times the cyclic frequency in rev/s. Thus,

$$\omega \;=\; 2\pi\text{ rad/rev} \times f \;=\; 2\pi\text{ rad/rev} \times 168\text{ rev/s} \;=\; 1056\text{ rad/s}, \tag{3}$$

or equivalently $\omega = 1056\text{ s}^{-1}$.

(c) The linear speed is $v = \omega \times r$ where r is the distance from the axis of rotation. For a point on the rim of the disk $r = R$ — the disk's radius, — hence

$$v \;=\; \omega \times R \;=\; 1056\text{ s}^{-1} \times 0.06\text{ m} \;=\; 63.3\text{ m/s} \;\approx\; 142\text{ miles/hour}. \tag{4}$$

(d) The centripetal acceleration is

$$a_c \;=\; \frac{v^2}{R} \;=\; \frac{(\omega \times R)^2}{R} \;=\; \omega^2 \times R \;=\; (1056\text{ s}^{-1})^2 \times 0.06\text{ m} \;=\; 66850\text{ m/s}^2 \tag{5}$$

or over 6800 times the free-fall acceleration g.

(e) For a point at distance $r = 3$ cm from the rotation axis, the linear speed of rotation is

$$v = r \times \omega = 0.03 \text{ m} \times 1056 \text{ s}^{-1} = 31.6 \text{ m/s} \approx 71 \text{ miles/hr} \qquad (6)$$

and the centripetal acceleration is

$$a_c = \frac{v^2}{r} = \omega^2 \times r = 33425 \text{ m/s} \approx 3410 \times g. \qquad (7)$$

Non-textbook problem #2:

(a) A solid disk or cylinder of mass M and radius R has moment of inertia $I = \frac{1}{2}MR^2$, *cf.* figure 7–11 of the textbook. Approximating the CD as a solid disk (*i.e.*, ignoring the small hole in the middle of the disk), we have

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2} \times 15 \text{ g} \times (6 \text{ cm})^2 = 270 \text{ g} \cdot \text{cm}^2 = 2.7 \cdot 10^{-5} \text{ kg} \cdot \text{m}^2. \qquad (8)$$

For your information, for a disk of (outer) radius R with a hole in the middle of a smaller radius r, the moment of inertia is $I = \frac{1}{2}M \times (R^2 + r^2)$. For a CD, the hole has radius 0.75 cm, thus taking it into account yields CD moment of inertia

$$I = \tfrac{1}{2} \times (15 \text{ g}) \times \left((6 \text{ cm})^2 + (0.75 \text{ cm})^2\right) = 274.22 \text{ g} \cdot \text{cm}^2. \qquad (9)$$

Comparing this result to the approximate formula (8) we see that the the approximation is about 1.5% off the mark.

(b) The angular momentum of a body with moment of inertia I spinning at angular frequency ω is $L = I\omega$. Thus, the angular momentum of the spinning CD is

$$L = I \times \omega = (2.7 \cdot 10^{-5} \text{ kg} \cdot \text{m}^2) \times (1056 \text{ s}^{-1}) = 0.0285 \text{ kg} \cdot \text{m}^2/\text{s}. \qquad (10)$$

(c) For the linear motion, the momentum changes with time at the rate equal to the net force, $\frac{\Delta P}{\Delta t} = F_{\text{net}}$. Likewise, for the rotation, the angular momentum changes with time at the rate equal to the net torque:

$$\frac{\Delta L}{\Delta t} \;=\; \tau_{\rm net}\,. \tag{11}$$

For rigid bodies $L = I \times \omega$ with constant moment of inertia I, hence $\Delta L = i \times \Delta\omega$ and therefore

$$\tau_{\rm net} \;=\; \frac{I\delta\omega}{\Delta t} \;=\; I \times \frac{\Delta\omega}{\Delta t} \;=\; I \times \alpha. \tag{12}$$

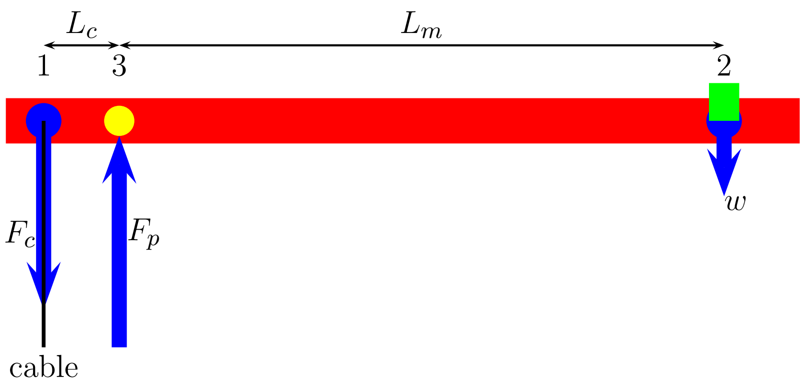
Note however that eq. (11) is more universal and applies to bodies of variable geometry — and hence variable I — as well as rigid bodies, but eq. (12) applies only to bodies of rigid geometry.

For the CD in question, the angular momentum increases from zero to $L = 0.0285\ {\rm kg\,m^2/s}$ in one second, thus the net torque acting on the CD must be

$$\tau \;=\; \frac{\Delta L}{\Delta t} \;=\; \frac{0.0285\ {\rm kg\,m^2/s}\;-\;0}{1\ {\rm s}} \;=\; 0.0285\ {\rm kg\,m^2/s^2} \;=\; 0.0285\ {\rm N\times m}. \tag{13}$$

Non-textbook problem #3:

The balancing part of the medical scales is subject to three forces: (1) the cable force F_c equal to the person's weight Mg; (2) the weight $w = mg$ of the $m = 1$ kg mass; and (3) a force F_p at the pivot point of the balance. These forces act are as follows:



The balance is pivoted at point 3 (the yellow dot on the picture), hence the force F_p acting at that point has zero lever arm and consequently does not generate any torque, $\tau_p = 0$.

The cable force acts at point 1, 1 cm left of the pivot, hence its lever arm is $L_c = 1$ cm; consequently, this force generates a torque of magnitude $\tau_c = F_c \times L_c = Mg \times L_c$ in the counterclockwise direction. Finally, the weight $w = mg$ acts at point 2 (the location of the $m = 1$ kg mass), 80 cm right of the pivot, thus its lever arm is $L_m = 80\ cm$ and the torque is $\tau_m = w \times L_m = mg \times L_m$ in the clockwise direction.

Putting all this torques together, we have the net torque

$$\tau_{\text{net}} \;=\; \tau_c \;+\; \tau_m \;+\; \tau_p \;=\; Mg \times L_c \;-\; mg \times L_m \;+\; 0. \tag{14}$$

When the balance is at equilibrium, this net torque must vanish, hence

$$Mg \times L_c \;-\; mg \times L_m \;=\; 0, \tag{15}$$

and therefore the person's mass M is equal to

$$M \;=\; \frac{m \times L_m}{L_c}\,. \tag{16}$$

Given $m = 1$ kg, $L_m = 80$ cm, and $L_c = 1$ cm, eq. (16) gives us $M = 80$ kg.

Non-textbook problem #4:

(a) The two astronaut system spins around its center of mass, located in the middle of the line. Thus, originally, each astronaut is at distance of R_1 from the center of rotation and the moment of inertia if the system is

$$I_1 \;=\; 2 \times M \times R_1^2 \;=\; 2 \times (100\text{ kg}) \times (10\text{ m})^2 \;=\; 20,000\text{ kg}\cdot\text{m}^2. \tag{17}$$

The system spins at frequency $f_1 = 3$ rev/min $= 0.05$ rev/s which corresponds to angular velocity $\omega_1 = 2\pi \times f_1 = 0.314\ rad/s$. Consequently, the net angular momentum is

$$L_1 \;= I_1 \times \omega_1 \;=\; 6280\text{ kg}\,\text{m}^2/\text{s}. \tag{18}$$

(b) To change the net angular momentum of the system one needs external torques and hence external forces. But the two astronauts are floating in space with their jet-packs off.

Although they are pulling each other via the line, this force is internal to the two-astronaut system and cannot change its net angular momentum. Hence, despite the changed geometry of the system — the astronauts are closer to the center of rotation than before — the angular momentum remains unchanged, $L_2 = L_1$.

The new configuration has astronauts closer to the center of rotation: $R_2 = 5\text{ m} = \frac{1}{2}R_1$. Consequently, the moment of inertia is reduced to

$$I_2 \;=\; 2\times M\times R_2^2 \;=\; 5,000\text{ kg}\cdot\text{m}^2 \;=\; \frac{1}{4}\times I_1. \tag{19}$$

But the angular momentum remains unchanged, so to compensate for the reduced I, the angular velocity ω has to increase:

$$\omega_2 \;=\; \frac{L_2}{I_2} \;=\; \frac{L_1}{\frac{1}{4}I_1} \;=\; 4\frac{L_1}{I_1} \;=\; 4\times\omega_1 \;=\; 1.256\text{ rad/s}. \tag{20}$$

Or in terms of the frequency

$$f_2 \;=\; \frac{\omega_2}{2\pi} \;=\; \frac{4\omega_1}{2\pi} \;=\; 4\times f_1 \;=\; 12\text{ rev/min}. \tag{21}$$