Textbook problem **8.13**:

(a) The potential energy of of a body in a gravitational field is

$$E_{\rm pot} \ = \ Mg \times h \tag{1}$$

where Mg is the body's weight and h is its altitude. For a softball of mass $M = 250$ g (and hence weight $Mg = 2.45$ N) at altitude $h = 15$ m, the potential energy is $E_{\rm pot} = 2.45\ \text{N} \times 15\ \text{m} = 36.75$ J.

(b) Let us assume no air resistance to the ball's flight, so the only force acting on the ball is gravity. Consequently, the ball's net mechanical energy is conserved,

$$E_{\rm mech} \ = \ E_{\rm kin} \ + \ E_{\rm pot} \ = \ \text{const.} \tag{2}$$

In the beginning, the ball has kinetic energy due to its speed but no altitude and hence no potential energy, thus

$$E_{\rm mech}^{\rm begin} \ = \ E_{\rm kin}^{\rm begin} \ + \ 0. \tag{3}$$

At the top of the trajectory, the ball's speed drops to zero, and so the kinetic energy, but now the ball has potential energy due to its altitude, thus

$$E_{\rm mech}^{\rm top} \ = \ 0 \ + E_{\rm pot}^{\rm top}\,. \tag{4}$$

But the net mechanical energy is conserved, which means that

$$E_{\rm mech}^{\rm begin} \ = \ E_{\rm mech}^{\rm top} \tag{5}$$

and hence according to eqs. (3) and (4),

$$E_{\rm kin}^{\rm begin} \ = \ E_{\rm pot}^{\rm top} \tag{6}$$

In part (a) we evaluated the potential energy of the ball at the top of its trajectory as $E_{\rm pot}^{\rm top} = 36.75$ J, hence according to eq. (6), the kinetic energy of the ball at the beginning of its flight has exactly the same value, $E_{\rm kin}^{\rm begin} = 36.75$ J.

In real life, there is air resistance and the net energy of the ball is not conserved. Instead, it keeps decreasing due to negative work of the air drag. Calculating this work is difficult because the drag force depends on the ball's speed, but we know this work is negative because the direction of the drag force is opposite to the direction of the ball's motion. Consequently, the initial net energy of the ball must be bigger than the net energy at the top, so if the flight tops at altitude 15 m, then the initial kinetic energy must be bigger than 36.75 Joules.

(c) Again we assume the air drag to be negligible, hence the net mechanical energy of the ball is conserved according to eq. (2). When the ball comes down and returns to your hand, it has zero potential energy, but it has non-zero speed and hence kinetic energy. Proceeding similar to part (b), we have

$$E_{\text{mech}}^{\text{end}} = E_{\text{kin}}^{\text{end}} + 0 = E_{\text{mech}}^{\text{top}} \tag{7}$$

and therefore

$$E_{\text{kin}}^{\text{end}} = E_{\text{pot}}^{\text{top}} = 36.75 \text{ J}. \tag{8}$$

And if the air resistance is important, then the ball keeps losing mechanical energy, so its kinetic energy at the and of its flight is less than 36.75 J, although the exact calculation is too difficult for this class.

(d) The kinetic energy of the ball is related to its speed according to

$$E_{\text{kin}} = \tfrac{1}{2}Mv^2 \implies v = \sqrt{\frac{2E_{\text{kin}}}{M}}\,. \tag{9}$$

At the beginning of the ball's flight, and again at its end, the ball has $E_{\text{kin}} = 36.75$ J and hence speed $v = 12.1$ m/s. Of course the direction of the ball's velocity is upward in the beginning of the flight, and downward at the end. Taking the upward direction to be positive, we have $v(\text{begin}) = +12.1$ m/s and $v(\text{end}) = -12.1$ m/s.

Textbook problem **8.16**:

(a) We don't know the downhill distance Bob is skiing, but we know his altitude decreases by 25 meters. Hence, his potential energy decreases according to

$$\Delta E_{\text{pot}} \;=\; Mg \times \Delta h \;=\; 65\text{ kg} \times 9.8\text{ m/s}^2 \times (-25\text{ m}) \;\approx\; -16000\text{ J}. \tag{10}$$

If the resistive forces (air drag and friction between the skis and the snow) are too small to matter, Ben's net mechanical energy is conserved, so the potential energy loss is the kinetic energy gain:

$$\Delta\left(E_{\text{mech}} = E_{\text{kin}} + E_{\text{pot}}\right) \;=\; 0 \;\Longrightarrow\; \Delta E_{\text{kin}} \;=\; -\Delta E_{\text{pot}} \;\approx\; +16000\text{ J}. \tag{11}$$

At the beginning of the run, Ben had zero speed and hence zero kinetic energy. During the run, E_{kin} increased by 16000 Joules, thus at the end $E_{\text{kin}} = 16000$ J.

(b) The speed is related to kinetic energy according to eq. (9), hence given the kinetic energy calculated in part (a), the speed is $v = \sqrt{2 \times 16000\text{ J}/65\text{ kg}} = 22$ m/s.

(c) In conventional units, the speed of 22 m/s is 80 km/hour or 50 miles per hour. The speed record for downhill skiing is over 140 MPH, so 50 MPH is quite reasonable for an experience skier, but it's dangerously fast for a novice on a "bunny slope". Fortunately, in real life one does not reach such speeds on a bunny slope because of resistive forces, especially the air drag.

Textbook problem **8.19**:

(a) Approximating the North American continent as a square slab of rock 5000 km on a side and 30 km deep gives us area $A = (5000\text{ km})^2 = 25 \cdot 10^6\text{ km}^2$ and volume $V = A \times \text{depth} = (25 \cdot 10^6\text{ km}^2) \times (30\text{ km}) = 750 \cdot 10^6\text{ km}^3$. Or in cubic meters, $V = 750 \cdot 10^6 \times (10^3\text{ m})^3 = 750 \cdot 10^6 \times 10^9\text{ m}^3 = 7.5 \cdot 10^{17}\text{ m}^3$. The mass of all this rock is the product of the volume and the density,

$$M \;=\; V \times \rho \;=\; (7.5 \cdot 10^{17}\text{ m}^3) \times (2800\text{ kg/m}^3) \;=\; 2.1 \cdot 10^{21}\text{ kg}. \tag{12}$$

(b) The continent has a very large mass but very low speed $v = 2$ cm/yr = (0.02 m)/(3.15 · 16^6 s) = $6.34 \cdot 10^{-10}$ m/s. Consequently, its kinetic energy

$$E_{\rm kin}^{NA} \;=\; \tfrac{1}{2}Mv^2 \;=\; \tfrac{1}{2}(2.1\cdot 10^{2}1\ {\rm kg})\times(6.34\cdot 10^{-10}\ {\rm m/s})^2 \;=\; 422\ {\rm J} \qquad (13)$$

has a rather everyday value.

(c) By comparison, a 70 kg jogger running at speed 5 m/s (18 km/hour or 11 MPH) has kinetic energy $E_{\rm kin}^{\rm jogger} = \frac{1}{2}(70\ {\rm kg})\times(5\ {\rm m/s})^2 = 875$ J. So the whole continent has only a half of the jogger's kinetic energy.

The non-textbook problem:

A horse of weight $Mg = 2000$ lb (the pound here is a unit of force rather than mass) walking up a hill of height $h = 1000$ feet increases his potential energy by

$$\Delta E_{\rm pot} \;=\; Mg\times h \;=\; 2\cdot 10^6\ {\rm ft\cdot lb}. \qquad (14)$$

This energy gain comes from the work performed by the horse himself, so his net mechanical work is $W = 2\cdot 10^6$ foot-pounds. Note that this work does not depend on the horizontal motion of the horse while he walks up the hill's slope, but only on his elevation gain.

Power is the ratio of work to the time it took,

$$P \;=\; \frac{W}{t}\,, \qquad (15)$$

so given the net amount of work and the power, we can calculate the time according to

$$t \;=\; \frac{W}{P}\,. \qquad (16)$$

For the problem at hand, the work is two million foot-pounds and the power is one horsepower or 550 foot-pounds per second, thus

$$t \;=\; \frac{2\cdot 10^6\ {\rm ft\cdot lb}}{550\ {\rm ft\cdot lb/s}} \;\approx\; 3600\ {\rm s} \;=\; 1\ {\rm hour}. \qquad (17)$$

In other words, given one horsepower worth of power, the horse heeds an hour to get to the top of the hill.

If you prefer metric unites, you can calculate the net work as

$$W = \Delta E_{\rm pot} = Mgh = 900 \text{ kg} \times 9.8 \text{ m/s}^2 \times 300 \text{ m} = 2.65 \cdot 10^6 \text{ J}, \tag{18}$$

and then identify the horsepower as 746 Watt, hence

$$t = \frac{W}{P} = \frac{2.65 \cdot 10^6 \text{ J}}{746 \text{ W}} \approx 3600 \text{ s} = 1 \text{ hour}. \tag{19}$$