Non-textbook problem **1**:

A food calorie is 4186 Joules of chemical energy. When human muscles use this energy, 25% of it — *i.e.*, 1046 J — is converted to mechanical energy while the rest is wasted as heat. Thus, "burning up" one food calorie produces 1046 J of mechanical energy.

Riding a bicycle at 20 km/hr requires 60 Watts of mechanical power. Over an hour of time, this amounts to

$$W \;=\; P\times t \;=\; 60\text{ W}\times 3600\text{ s} \;=\; 216000\text{J} \tag{1}$$

of mechanical work. And to produce this work, one needs to burn

$$Q \;=\; \frac{216000\text{ J}}{1046\text{ J/kcal}} \;=\; 206\text{ kcal} \tag{2}$$

worth of food (or body fat).

Non-textbook problem **2**:

The key to this question is the molecular weight of sucrose. Summing over the atoms comprising one sucrose molecule, we find

$$\begin{aligned}\mu(C_{12}H_{22}O_{11}) \;&=\; 12\times\mu(C) \;+\; 22\times\mu(H) \;+\; 11\times\mu(O)\\ &=\; 12\times 12 \;+\; 22\times 1 \;+\; 11\times 16\\ &=\; 342.\end{aligned} \tag{3}$$

This molecular weight means that the mass of 1 molecule of sucrose is 342 atomic units, and also that the mass of 1 mol of sucrose is 342 grams. Consequently, one pound or 454 grams of sugar contains

$$n \;=\; \frac{M}{\mu} \;=\; \frac{454\text{ g}}{342\text{ g/mol}} \;=\; 1.33\text{ mol} \tag{4}$$

of sucrose. One mol of a substance contains $N_A = 6.02\cdot 10^{23}$ molecules, hence 1.33 mols of sucrose in one pound of sugar contain

$$N = n \times N_A = 1.33 \times 6.02 \cdot 10^{23} \approx 8 \cdot 10^{23} \text{ molecules.} \qquad (5)$$

Non-textbook problem **3**:

When planet Earth was formed 4.5 billion years ago, it contained two copper isotopes ${}^{63}Cu$ and ${}^{65}Cu$ in some primordial ratio. The isotopes were well mixed, and no natural process on Earth could separate them, so the ratio remains fixed. In other words, all natural copper on Earth has the same fractions f_{63} and f_{65} of the two isotopes. By "natural" here I mean all copper in all its form and compounds, naturally occurring or made by industry, as long as it hasn't been through a nuclear lab which separated isotopes. One mol of natural copper contains f_{63} mol of ^{63}Cu and f_{65} mol of ^{65}Cu, so its net mass in grams is

$$\bar{\mu}(\text{natural Cu}) = f_{63} \times \mu({}^{63}\text{Cu}) + f_{65} \times \mu({}^{65}\text{Cu}). \qquad (6)$$

In other words, the atomic weight of the natural copper is given by eq. (6).

In this problem we are given the atomic weights of the two isotopes as well as the atomic weight of natural copper, but we do not know the isotope fractions f_{63} and f_{65}. However, since there are only two isotopes, we know that

$$f_{63} + f_{65} = 1. \qquad (7)$$

Together, eqs. (6) and (7) give us two linear equations for two unknown variables f_{63} and f_{65}. The rest is algebra.

To solve the equations, we first rewrite eq. (7) as

$$f_{63} = 1 - f_{65} \qquad (8)$$

and then substitute this formula into eq. (6):

$$\begin{aligned}
\bar{\mu}(\text{natural Cu}) &= (1-f_{65})\times\mu(^{63}\text{Cu}) \;+\; f_{65}\times\mu(^{65}\text{Cu}) \\
&\downarrow \\
\bar{\mu}(\text{natural Cu}) \;-\; \mu(^{63}\text{Cu}) &= f_{65}\times\big(\mu(^{65}\text{Cu}) \;-\; \mu(^{63}\text{Cu})\big) \\
&\downarrow \\
f_{65} &= \frac{\bar{\mu}(\text{natural Cu}) \;-\; \mu(^{63}\text{Cu})}{\mu(^{65}\text{Cu}) \;-\; \mu(^{63}\text{Cu}}\,.
\end{aligned} \tag{9}$$

Evaluating this formula, we have

$$f_{65} \;=\; \frac{63.55 \;-\; 62.93}{64.93 \;-\; 62.93} \;=\; \frac{0.62}{2.00} \;=\; 0.31, \tag{10}$$

and hence by eq. (7)

$$f_{63} \;=\; 1 \;-\; 0.31 \;=\; 0.69. \tag{11}$$

In other words, natural copper contains 69% of ^{63}Cu and 31% of ^{65}Cu.

Non-textbook problem **4**:

(a) When two atoms of deuterium fuse into one atom of helium–4, there is a small loss of mass. Specifically,

$$\delta m \;=\; 2\times 2.0141\text{ a.m.u.} \;-\; 1\times 4.0026\text{ a.m.u.} \;=\; 0.0256\text{ a.m.u.} \tag{12}$$

This mass loss is $\frac{0.0256\text{ amu}}{4.028\text{ amu}} \approx 0.64\%$ of the of the initial deuterium mass. So when a whole kilogram of deuterium fuses to helium, the mass loss amounts to $\Delta m = 0.64\% \times 1\text{ kg} = 6.4\text{ g}$. That is, out of 1000 g of deuterium we get 993.6 g of helium, but the remaining 6.4 g are converted to energy according to Einstein formula

$$E \;=\; \Delta m \times c^2 \;=\; (0.0064\text{ kg}) \times (3\cdot 10^8\text{ m/s})^2 \;=\; 5.76\cdot 10^{14}\text{ J}. \tag{13}$$

(b) One liter of seawater contains about 35 milligrams of deuterium. If we could fuse that deuterium in a controlled manner, we would get

$$35 \text{ mg} \times \frac{5.76 \cdot 10^{14} \text{ J}}{1 \text{ kg}} \;=\; 35 \cdot 10^{-6} \times 5.76 \cdot 10^{14} \text{ J} \;\approx 2 \cdot 10^{10} \text{ Joules} \tag{14}$$

worth of energy. By comparison, burning 1 liter of gasoline yields only 35 MJ — *i.e.*, $35 \cdot 10^6$ Joules — of energy. Thus, to match the energy yield of fusing deuterium in 1 liter of seawater we would need

$$\frac{2 \cdot 10^{10} \text{ J}}{35 \cdot 10^6 \text{ J/L}} \;\approx 570 \text{ liters} \tag{15}$$

of gasoline.