

Non-textbook problem 1:

Since the ocean is open to the atmosphere, water pressure at depth D is given by

$$P(D) = P_{\text{atm}} + \rho(\text{water}) \times g \times D. \quad (1)$$

At the bottom of the Challenger Deep, $D = 10923$ m and

$$\rho \times g \times D = 1050 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 10923 \text{ m} = 1.124 \cdot 10^8 \text{ N/m}^2 = 1.124 \cdot 10^8 \text{ Pa}. \quad (2)$$

Adding the atmospheric pressure $P_{\text{atm}} = 1.013 \cdot 10^5$ Pa, we find the pressure at the deepest place in the ocean

$$P = 1.125 \cdot 10^8 \text{ Pa} = 1125 \text{ bar} = 1110 \text{ atm} = 16320 \text{ PSI}, \quad (3)$$

or a bit over 8 tons per square inch.

Non-textbook problem 2:

Atmospheric pressure is related to the atmosphere's own weight per unit of area. The pressure of 101300 Pascal means that the air above one square meter of Earth's surface pushes the surface down with the force of 101300 Newtons; by Newton's third law, the surface pushes back, and this upward force balances the weight of the air. Thus, the air over one square meter weighs 10130 Newtons. Translating this weight into mass (via dividing by g) gives $101300/9.8 = 10340$ kg of air over each square meter of surface. And over the whole Earth's surface of area $A = 4\pi R_E^2 = 511 \cdot 10^6 \text{ km}^2 = 511 \cdot 10^{12} \text{ m}^2$, the total mass of the air is

$$M = \frac{P}{g} \times A = 10340 \text{ kg/m}^2 \times 511 \cdot 10^{12} \text{ m}^2 = 5.28 \cdot 10^{18} \text{ kg}. \quad (4)$$

This is approximately one millionth of the total mass of the planet.

Non-textbook problem 3:

Weighing the bowl in the air gives its true weight $W^{\text{true}} = Mg$. But when the bowl is weighed under water, its apparent weight is reduced by the buoyant force, thus

$$\begin{aligned} W^{\text{app}} &= W^{\text{true}} - F^{\text{buoyant}} \\ &= Mg - V \times \rho_{\text{water}} \times g \end{aligned} \tag{5}$$

where V is the volume of water displaced by the bowl. When the bowl is completely submerged, the empty space inside the bowl is filled by water, so only the glass displaces the water, not the empty space. Which means that V is simply the volume occupied by glass, which is related to the bowl's mass according to

$$V = \frac{M}{\rho_{\text{glass}}}. \tag{6}$$

Consequently,

$$W^{\text{app}} = Mg - \frac{M}{\rho_{\text{glass}}} \times \rho_{\text{water}} \times g = Mg \times \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{glass}}} \right). \tag{7}$$

Or in other words,

$$\frac{W_{\text{in water}}^{\text{app}}}{W_{\text{in air}}} = 1 - \frac{\rho_{\text{water}}}{\rho_{\text{glass}}}. \tag{8}$$

Inverting this formula, we have

$$\begin{aligned} \frac{\rho_{\text{water}}}{\rho_{\text{glass}}} &= 1 - \frac{W_{\text{in water}}^{\text{app}}}{W_{\text{in air}}} \\ \langle\langle \text{for the bowl in question} \rangle\rangle & \\ &= 1 - \frac{3 \text{ lb}}{5 \text{ lb}} = 1 - 0.6 = 0.4, \end{aligned} \tag{9}$$

and consequently

$$\rho_{\text{glass}} = \frac{\rho_{\text{water}}}{0.4} = \frac{1 \text{ g/cm}^3}{0.4} = 2.5 \text{ g/cm}^3 = 2500 \text{ kg/m}^3. \tag{10}$$

Non-textbook problem 4:

For simplicity, suppose the glass is a perfect cylinder of horizontal section $A = \pi R^2$. Then the water level in the glass is given by

$$H = \frac{1}{A} \times (V_{\text{liq}} + V_{\text{disp}}) \quad (11)$$

where V_{liq} is the volume of liquid water in the glass, and V_{disp} is the volume of water displaced by the ice cube. By Archimedes's law, the latter volume governs the buoyant force on the cube, and since the cube floats without sinking or rising, this buoyant force must be equal to the cube's weight:

$$V_{\text{disp}} \times \rho_{\text{liq.water}} \times g = M_{\text{ice}} \times g, \quad (12)$$

and therefore

$$V_{\text{disp}} = \frac{M_{\text{cube}}}{\rho_{\text{liq.water}}}. \quad (13)$$

Note that this formula involves the density of the liquid water rather than ice because we don't want the volume of the whole cube but only of its submerged part which displaces water.

Combining eq. (13) with an obvious formula

$$V_{\text{liq}} = \frac{M_{\text{liq}}}{\rho_{\text{liq.water}}}, \quad (14)$$

we find

$$V_{\text{liq}} + V_{\text{disp}} = \frac{M_{\text{liq}} + M_{\text{ice}}}{\rho_{\text{liq.water}}} = \frac{M_{\text{total H}_2\text{O}}}{\rho_{\text{liq.water}}} \quad (15)$$

and hence via eq. (11),

$$H = \frac{M_{\text{total H}_2\text{O}}}{A \times \rho_{\text{liq.water}}}. \quad (16)$$

In other words, the water level depends only on the total mass of H_2O in both liquid and solid form, and it does not matter how much H_2O is frozen and how much is liquid, as long as all the ice floats on liquid water. Therefore, when the ice cube melts and turns into more liquid water, the water level stays exactly the same because the total mass of H_2O remain unchanged.

Now suppose the glass is not cylindrical but has a different shape. In this case, the water level is a more complicated function of $V_{\text{liq}} + V_{\text{disp}}$, but it's still a function of the same sum of liquid volume and displaced volume. According to eq. (15), this sum remains unchanged when the ice cube melts, hence regardless of the glass's shape, the water level remains unchanged.