Non-textbook problem **1**:

(a) The continuity equation says that the flow rate

$$\mathcal{F} \;\equiv\; A \times v \;=\; \text{const} \tag{1}$$

is constant all along the pipe. Consequently, the water speeds up when the pipe narrows down and slows down when the pipe widens up. Comparing the two segments of the pipe — one wide and the other narrow — we can relate the speed of water through the two segments as

$$\frac{v_{\text{narrow}}}{v_{\text{wide}}} \;=\; \frac{A_{\text{wide}}}{A_{\text{narrow}}}\,. \tag{2}$$

For the problem at hand, the wider segment has twice the *diameter* of the narrow segment, and therefore *four times* the cross-sectional area. Indeed, for a cylindrical pipe of diameter d,

$$A \;=\; \pi R^2 \;=\; \pi(d/2)^2 \;=\; \frac{\pi}{4} \times d^2 \tag{3}$$

and therefore

$$\frac{A_{\text{wide}}}{A_{\text{narrow}}} \;=\; \left(\frac{d_{\text{wide}}}{d_{\text{narrow}}}\right)^2 \;=\; (2\ \text{cm}/1\ \text{cm})^2 \;=\; 2^2 \;=\; 4. \tag{4}$$

Hence, according to eq. (2),

$$\frac{v_{\text{narrow}}}{v_{\text{wide}}} \;=\; 4 \;\Longrightarrow\; v_{\text{narrow}} \;=\; 4 \times v_{\text{wide}} \;=\; 40\ \text{m/s}. \tag{5}$$

(b) According to Bernoulli equation

$$P \;+\; \tfrac{1}{2}\rho v^2 \;+\; \rho g h \;=\; \text{const}. \tag{6}$$

For a horizontal pipe the $\rho g h$ term is itself constant, hence

$$P \;+\; \tfrac{1}{2}\rho v^2 \;=\; \text{const}, \tag{7}$$

which means that for the two pipe segments

$$P_{\text{wide}} \;+\; \frac{\rho}{2}\times v_{\text{wide}}^2 \;=\; P_{\text{narrow}} \;+\; \frac{\rho}{2}\times v_{\text{narrow}}^2. \tag{8}$$

Consequently,

$$\begin{aligned} P_{\text{wide}} \;-\; P_{\text{narrow}} \;&=\; \frac{\rho}{2}\times\left(v_{\text{narrow}}^2 \;-\; v_{\text{wide}}^2\right) \\ &=\; \frac{1000\ \text{kg/m}^3}{2}\times\left((40\ \text{m/s})^2 \;-\; (10\ \text{m/s})^2\right) \\ &=\; 750000\ \text{Pa} \;=\; 750\ \text{kPa} \;\approx\; 7.5\ \text{atm}. \end{aligned} \tag{9}$$

And since the narrow segment of the pipe opens to the air, the pressure there is equal to the atmospheric pressure, $P_{\text{narrow}} = 1\ \text{atm} \approx 100\ \text{kPa}$, it follows that the pressure in the wide segment is $P_{\text{wide}} = 100\ \text{kPa} + 750\ \text{kPa} = 850\ \text{kPa} \approx 8.5\ \text{atm} \approx 124\ \text{PSI}$.

Or to be precise, the *absolute* water pressure in the wide segment is 850 kPa (8.5 atm or 124 PSI), while the *gauge* pressure (the difference between the absolute water pressure and the air pressure outside the pipe) is 750 kPa (7.5 atm, or 109 PSI).

Non-textbook problem **2**:

Thermal expansion of most solid materials is linear, which means that all lengths change with temperature according to

$$\Delta L \;=\; L_0\times\alpha\times\Delta T \tag{10}$$

For steel, the coefficient α of thermal expansion is $\alpha = 6.5\cdot 10^{-6}/°F$, hence for temperature changes between $0°F$ and $100°F$,

$$\frac{\delta L}{L_0} \;=\; \alpha\times\Delta T \;=\; (6.5\cdot 10^{-6}/°F)\times(100°F \;-\; 0°F) \;=\; 6.5\cdot 10^{-4}, \tag{11}$$

which means every meter of steel changes its length by $6.5\cdot 10^{-4}\ \text{m} = 0.65\ \text{mm}$. And for the $L_0 = 150$ m steel rail, this thermal expansion amounts to

$$\Delta L \;=\; 150\ \text{m}\times 6.5\cdot 10^{-4} \;=\; 9.75\ \text{cm}. \tag{12}$$

In other words, the rail is almost 10 cm (4 inches) longer in summer than it is in winter.

Non-textbook problem **3**:

The ideal gas law says that for a fixed amount of gas,

$$\frac{P \times V}{T^{\rm abs}} \;=\; \text{const.} \tag{13}$$

Consequently, when the temperature and pressure of helium gas in the balloon change with the balloon's altitude, the volume of the gas changes according to

$$\frac{P \times V}{T^{\rm abs}} \;=\; \frac{P_0 \times V_0}{T_0^{\rm abs}} \;\Longrightarrow\; V \;=\; V_0 \times \frac{T^{\rm abs}}{T_0^{\rm abs}} \bigg/ \frac{P}{P_0} \tag{14}$$

Note that the temperatures in these formulæ are *absolute* temperatures, measured from absolute zero in degrees Kelvin (or Rankin). So let's translate both T and T_0 into the Kelvin scale using

$$T(\text{in}\,^\circ F) \;=\; \frac{9}{5} \times T(\text{in}\,^\circ C) \,+\, 32 \;\Longrightarrow\; T(\text{in}\,^\circ C) \;=\; \frac{5}{9} \times \Big(T(\text{in}\,^\circ F) \,-\, 32\Big), \tag{15}$$

and

$$T^{\rm abs}(\text{in}\,^\circ K) \;=\; T(\text{in}\,^\circ C) \;+\; 273.15. \tag{16}$$

Thus, $T_0 = 77^\circ F$ on the ground translates into $T_0 = 25^\circ C$ and hence $T_0^{\rm abs} = 298^\circ K$, while $T = 23^\circ F$ at 10 000 ft altitude translates into $T = -5^\circ C$ and hence $T^{\rm abs} = 268^\circ K$.

Substituting these temperatures into eq. (14) we find that the balloon expands to

$$V \;=\; (100\ \mathrm{m}^3) \times \frac{268^\circ\mathrm{K}}{298^\circ\mathrm{K}} \bigg/ \frac{690\ \mathrm{mbar}}{1000\ \mathrm{mbar}} \;=\; (100\ \mathrm{m}^3) \times 0.901/0.69 \;=\; 130.6\ \mathrm{m}^3. \tag{17}$$

Non-textbook problem **4**:

According to the universal gas law,

$$\frac{P \times V}{T} \;=\; n \times R \tag{18}$$

where V is the volume of the gas, P is its pressure, T is its absolute temperature, $R = 8.314\ \mathrm{J}/^\circ\mathrm{K/mol} = 8314\ \mathrm{J}/^\circ\mathrm{K/kmol}$ is a universal gas constant, same for all gases, and n is the amount of gas in mols or kilomols. In terms of the mass M of the gas,

$$n = \frac{M}{\mu} \tag{19}$$

where μ is gas's molecular weight; in this formula, if m is in grams then n is in mols, and if M is in kilograms then n is in kilomols. Combining eqs. (18) and (19) we obtain

$$\frac{P \times V}{T} = \frac{M}{\mu} \times R, \tag{20}$$

hence

$$M = \frac{\mu \times P \times V}{R \times T} \tag{21}$$

and therefore density

$$\rho \equiv \frac{M}{V} = \frac{\mu \times P}{R \times T} \tag{22}$$

For the carbon dioxide CO_2, $\mu = 12 + 2 \times 16 = 44$, which means one molecule's mass is 44 atomic units, one mol's mass is 44 grams, and one kilomols's mass is 44 kilograms. Hence under conditions prevailing on the surface of Venus — pressure $P = 92$ bar $= 9.2 \cdot 10^6$ Pa $=$ $9.2 \cdot 10^6$ J/m^3, and absolute temperature $T = 740°K$ — the CO_2 has density

$$\rho = \frac{(44\ \mathrm{kg/kmol}) \times (9.2 \cdot 10^6\ \mathrm{J/m^3})}{(8314\ \mathrm{J/kmol/°K}) \times (740°\mathrm{K})} = 65.8\ \mathrm{kg/m^3}. \tag{23}$$

Non-textbook problem **5**:

Initially, the wort is at $T_w^0 = 35°$C and the rocks are at $T_r^0 = 360°$C. After the rocks and the wort reach thermal equilibrium, they all have the same temperature T^e — and that's what we need to calculate.

During the process (of reaching the equilibrium) the rocks cool down from T_r^0 to T^e, which releases the amount of heat given by

$$Q_r \;=\; C_r \times (T_r^0 \;-\; T^e) \tag{24}$$

where

$$C_r \;=\; M_r \times c_r \;=\; 20 \text{ kg} \times 0.19 \text{ cal/g/°C} \;=\; 3.8 \text{ kcal/°C} \tag{25}$$

is heat capacity of the rocks. At the same time, the wort warms up from T_w^0 to T^e, which consumes the amount of heat given by

$$Q_w \;=\; C_w \times (T^e \;-\; T_w^0) \tag{26}$$

where

$$C_w \;=\; M_w \times c_w \;=\; 20 \text{ kg} \times 1.04 \text{ cal/g/°C} \;=\; 20.8 \text{ kcal/°C}; \tag{27}$$

is heat capacity of the wort. This heat comes from the cooling rocks, thus $Q_w = Q_r$ and therefore

$$C_w \times (T^e \;-\; T_w^0) \;=\; C_r \times (T_r^0 \;-\; T^e). \tag{28}$$

Solving this equation for the equilibrium temperature T^e, we find

$$T^e \times (C_w \;+\; C_r) \;=\; T_w^0 \times C_w \;+\; T_r^0 \times C_r \tag{29}$$

and therefore

$$\begin{aligned}
T^e \;&=\; \frac{C_w}{C_w + C_r} \times T_w^0 \;+\; \frac{C_r}{C_w + C_r} \times T_r^0 \\
&=\; \frac{20.8 \text{ kcal/°C}}{20.8 \text{ kcal/°C} \;+\; 3.8 \text{ kcal/°C}} \times 35\text{°C} \;+\; \frac{3.8 \text{ kcal/°C}}{20.8 \text{ kcal/°C} \;+\; 3.8 \text{ kcal/°C}} \times 360\text{°C} \\
&=\; 0.8455 \times 35\text{°C} \;+\; 0.1545 \times 360\text{°C} \\
&=\; 85.2\text{°C}.
\end{aligned} \tag{30}$$

Non-textbook problem **6**:

(a) A 100 g bullet moving at speed 400 m/s has kinetic energy

$$E \;=\; \tfrac{1}{2}M \times v^2 \;=\; \tfrac{1}{2} \times 0.1\ \text{kg} \times (400\ \text{m/s})^2 \;=\; 8000\ \text{J}. \tag{31}$$

When the bullet hits the target this energy is converted to heat; the amount of heat $Q = E = 8000$ J, or in calories

$$Q \;=\; 8000\ \text{J} \Big/ (4.186\ \text{J/cal}) \;=\; 1911\ \text{cal}. \tag{32}$$

The ice is already at the melting point of 0°C, hence the amount of heat needed for melting is simply the latent heat of fusion $L_f = 80$ cal/g, *i.e.*

$$Q \;=\; L_f \times m \tag{33}$$

where m is the mass of melted ice. (Note $m < M = 100$ g.) Thus,

$$m \;=\; \frac{Q}{L_f} \;=\; \frac{1911\ \text{cal}}{80\ \text{cal/g}} \;\approx\; 24\ \text{g}. \tag{34}$$

The remaining 76 g if ice remain un-melted.

(b) To completely vaporize the bullet, we need to melt the ice, warm up the water from 0°C to 100°C, and then vaporize the water. The net amount of heat required for this process is at least

$$\begin{aligned} Q_{\text{min}} \;&=\; M \times L_f \;+\; M \times c \times (100^\circ\text{C} \;-\; 0^\circ\text{C}) \;+\; M \times L_v \\ &=\; M \times [80\ \text{cal/g} \;+\; 1\ \text{cal/g/}^\circ\text{C} \times 100^\circ\text{C} \;+\; 540\ \text{cal/g}] \\ &=\; M \times 720\ \text{cal/g}. \end{aligned} \tag{35}$$

Note that this is the minimal amount; for $Q > Q_{\text{min}}$, the whole bullet vaporizes, and then the excess heat goes to increase the vapor's temperature above the boiling point 100°C.

In energy units,

$$\frac{Q_{\text{min}}}{M} \;=\; 720\ \text{cal/g} \times 4.186\ \text{J/cal} \;=\; 3014\ \text{J/g} \;=\; 3.014 \cdot 10^6\ \text{J/kg}. \tag{36}$$

This means that if this heat comes from the kinetic energy of the bullet, we need at least 3.014 Megajoules of kinetic energy per kilogram of bullet's mass. But

$$E_{\text{kin}} \;=\; \tfrac{1}{2}Mv^2 \;\Longrightarrow\; \frac{E_{\text{kin}}}{M} \;=\; \tfrac{1}{2}v^2, \tag{37}$$

hence minimal kinetic energy per mass implies minimum speed v_{min} such that

$$\tfrac{1}{2}v_{\text{min}}^2 \;=\; \frac{E_{\text{kin}}^{\text{min}}}{M} \;=\; 3.104 \cdot 10^6\ \text{J/kg} \;\Longrightarrow\; v_{\text{min}} \;=\; \sqrt{2 \times 3.104 \cdot 10^6\ \text{J/kg}} \;=\; 2455\,\text{m/s}. \tag{38}$$

Note that this minimal speed does not depend on the bullet's mass: Any amount of ice — from a snowflake to an icy asteroid — will be completely vaporized on impact if it hits a hard target at this speed or faster.