Problem **1**:

Compare orbit equations for Ariel and for Luna:

$$T_A = 2\pi\sqrt{\frac{R_A^3}{GM_U}}\,, \qquad T_L = 2\pi\sqrt{\frac{R_L^3}{GM_E}}\,, \tag{1}$$

where M_U is Uranus's mass and M_E is Earth's mass. Taking the ratio of the two periods, we have

$$\frac{T_A}{T_L} = \frac{\sqrt{R_A^3/GM_U}}{\sqrt{R_L^3/GM_E}} \tag{2}$$

and therefore

$$\left(\frac{T_A}{T_L}\right)^2 = \frac{R_A^3/GM_U}{R_L^3/GM_E} = \left(\frac{R_A}{R_L}\right)^3 \Bigg/ \left(\frac{M_U}{M_E}\right). \tag{3}$$

Solving this equation for the mass ratio, we obtain

$$\begin{aligned} \frac{M_U}{M_E} &= \left(\frac{R_A}{R_L}\right)^3 \Bigg/ \left(\frac{T_A}{T_L}\right)^2 \\ &= \left(\frac{1}{2}\right)^3 \Bigg/ \left(\frac{1}{11}\right)^2 = \frac{1}{8} \Bigg/ \frac{1}{121} = \frac{121}{8} \approx 15. \end{aligned} \tag{4}$$

That is, Uranus is 15 times more massive than Earth.

Problem **2**:

The net momentum of two colliding boats is conserved during the collision:

$$M_A v_A^0 + M_B v_B^0 = M_A v_A' + M_B v_B' \tag{5}$$

where M_A and M_B are gross masses of Andy's and Bob's boats, v_A^0 and v_B^0 are their velocities immediately before the collision, and v_A' and v_B' are their velocities immediately after the collision. Since the collision is inelastic, $v'_A = v'_B$. Consequently, we may solve for $v' = v'_A = v'_B$:

$$v' = \frac{M_A v_A^0 + M_B v_B^0}{M_A + M_B} = \frac{M_A}{M_A + M_B} \times v_A^0 + \frac{M_B}{M_A + M_B} \times v_B^0 . \tag{6}$$

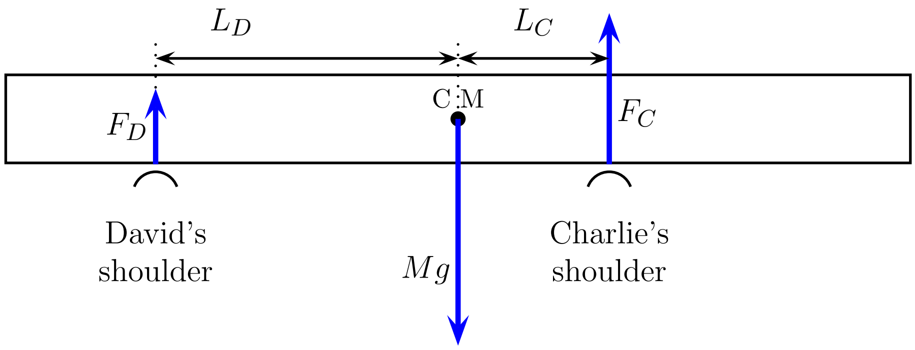
Numerically, $M_A = 1000$ lb, $M_B = 2000$ lb, $v_A^0 = +35$ MPH, and $v_B^0 = -25$ MPH; note opposite signs of the two velocities. Consequently,

$$\begin{aligned} v' &= \frac{1000\,\text{lb}}{1000\,\text{lb} + 2000\,\text{lb}} \times (+35\ \text{MPH}) + \frac{2000\,\text{lb}}{1000\,\text{lb} + 2000\,\text{lb}} \times (-25\ \text{MPH}) \\ &= \frac{1}{3} \times 35\ MPH - \frac{2}{3} \times 25\ MPH = -5\ MPH. \end{aligned} \tag{7}$$

That is, the velocity of the two-boat wreck immediately after the collision is 5 MPH is the Southward direction.

Problem **3**:

Let us picture the beam and the forces acting on it:



The net force on the beam must vanish, $Mg - F_D - F_C = 0$, and this allows as to use any point we like as a pivot for calculating the net torque. So let's use David's shoulder as a pivot. With this choice, beam's weight Mg has lever arm $= L_D$ and hence torque $\tau_W = Mg \times L_D$ in the clockwise direction. The force F_C from Charlie's shoulder has lever arm $= L_D + L_C$ and hence torque $\tau_C = -F_C \times (L_D + L_C)$; the sign is negative because this torque is counterclockwise. Finally, the force F_D from David's shoulder has zero lever arm and hence zero torque, $\tau_D = 0$. Altogether, the net torque is

$$\tau_{\text{net}} \;=\; \tau_W \;+\; \tau_C \;+\; \tau_D \;=\; Mg \times L_D \;-\; F_C \times (L_D + L_C) \;+\; 0. \tag{8}$$

In equilibrium, this net torque must vanish, and therefore

$$F_C \;=\; \frac{Mg \times L_D}{L_D + L_C} \;=\; Mg \times \frac{L_D}{L_D + L_C} \;=\; 90\text{ lb} \times \frac{4\,\text{ft}}{4\,\text{ft} \,+\, 2\,\text{ft}} \;=\; 60\text{ lb}. \tag{9}$$

Thus, Charlie's shoulder pushes the beam up with a 60 pound force, or in other words, Charlie carries 60 pounds of the beam's weight.

As to the weight on David's shoulder, we use $Mg - F_C - F_D = 0$ to obtain $F_D = Mg - F_C = 90\text{ lb} \,-\, 60\text{ lb} = 30\text{ lb}$.

Alternatively, we may re-calculate the torques using Charlie's shoulder as a pivot point. This gives zero lever arm for Charlie's force F_C and hence $\tau'_C = 0$; the beams weight Mg now has lever arm $= L_C$ and hence the torque $\tau'_W = -Mg \times L_C$ (the '$-$' sign indicates counterclockwise direction); and for the David's force F_D the lever arm is $L_C + L_D$ and the torque $\tau'_D = +F_D \times (L_C + L_D)$. Consequently, the net torque is

$$\tau'_{\text{net}} \;=\; \tau'_C \;+\; \tau'_W \;+\; \tau'_D \;=\; 0 \;-\; Mg \times L_C \;+\; F_D \times (L_C + L_D). \tag{10}$$

Again, we demand that this torque must vanish (since the beam is in equilibrium), which gives us

$$F_D \;=\; \frac{Mg \times L_C}{L_C + L_D} \;=\; Mg \times \frac{L_C}{L_C + L_D} \;=\; 90\text{ lb} \times \frac{2\,\text{ft}}{2\,\text{ft} \,+\, 4\,\text{ft}} \;=\; 30\text{ lb}. \tag{11}$$

Thus, David's shoulder carries 30 pounds of the beam's weight.

PS: In this problem, we use pound (lb) as a unit of force rather than mass. The beam weighs 90 pounds, which means $Mg = 90$ lb.

Problem **4**:

Mechanical work is the product W = force $\times$ displacement *in the direction of the force*. The force $\mathbf{F}$ in question is provided by student's legs. It balances the student's weight $M\mathbf{g}$, hence the magnitude $F = Mg = 60$ kg $\times$ 9.8 m/s$^2 \approx 600$ N, and the direction of $\mathbf{F}$ is vertically up.

Consequently, the *displacement in the direction of the force* $\mathbf{F}$ is the *vertical* component of the student's displacement, *i.e.* his elevation gain $\Delta h = 10$ m. The horizontal motion of the student is perpendicular to the force $\mathbf{F}$ and does not count towards the mechanical work. Thus, $W = F \times \Delta h = 600$ N $\times$ 10 m $= 6000$ Joules.