This homework is primarily a reading assignment. Specifically, you should study section §4.5 of the *Peskin & Schroeder* textbook about scattering cross sections and unstable particles' decay rates and their relations to the transition matrix elements $\mathcal{M}$(initial particles $\to$ final particles).

In class, I shall explain where such matrix elements come from and we shall spend the rest of the semester learning how to calculate them in perturbation theory. Unfortunately, I do not have enough class-time to explain how these matrix elements govern particles' scattering and decay processes, hence this assignment.

In addition to the reading assignment, I ask you to do a simple exercise as a practical application of the material you read:

In Fall of 2000, CERN lab in Geneva shut down LEP II — the second-stage Large Electron–Positron collider — to make room for the LHC proton-antiproton collider (finished this summer, and now they try to make it work). Just before its final shutdown, LEP II saw a few events that could be the first evidence of the Higgs particle being produced in e^+e^- collisions — but unfortunately not enough of these events to decide whether they really involved the Higgs particle. The experimental discovery of the Higgs and the measurement of its mass is therefore postponed until the LHC starts working and gets enough data.

This exercise is a part of the theoretical calculation of the Higgs production rate at an electron-positron collider such as LEP. Specifically, consider the

$$e^+ \ + \ e^- \ \to \ H^0 \ + \ Z^0 \tag{1}$$

process in which the positron and the electron annihilate into the Higgs particle (spin=0) and the Z^0 particle (spin=1). The Z^0 particle (one of the three mediators of the weak force, the other two being $W^\pm$) has mass $M_Z \approx 91\ \mathrm{GeV}/c^2$; the mass of the Higgs particle is currently unknown but is believed to be heavier — but not too much heavier — than M_Z; the initial electron and positron are so ultra-relativistic that we may treat them as massless, $M_e \approx 0$.

The Standard Model yields a simple formula for the transition process (1). Or rather, *to the leading order of the perturbation theory* there is a simple formula for the matrix element,

$$\mathcal{M}(e^+ + e^- \to H^0 + Z^0) \;=\; \frac{e^2 M_Z}{4\sin^2\theta_W}\,\frac{1}{s - M_Z^2} \times \epsilon_\mu(Z^0)\,\bar{v}(e^+)(4\sin^2\theta_W - 1 + \gamma^5)\gamma^\mu u(e^-) \tag{2}$$

where

$$s \;=\; (p_{e^+} + p_{e^-})^2 \;=\; (p_Z + p_H)^2 \;=\; E_{\text{c.m.}}^2\,, \tag{3}$$

$\epsilon_\mu(Z^0)$ is the polarization vector of the final Z^0 vector particle, $\bar{v}(e^+)$ and $u(e^-)$ are the polarization spinors of the initial positron and electron, θ_W is the electroweak mixing angle, $\sin^2\theta_W \approx 0.233$, and the electric charge e is in the rationalized $\hbar = c = 1$ units, $\alpha_{EM} = e^2/(4\pi) \approx 1/137$.

Later this semester, we shall learn how to average the $|\mathcal{M}|^2$ over the spin states of the initial electron and positron and sum over the spin states of the final Z^0 particle, but for the purpose of this exercise, let me simply give you the result:

$$\frac{1}{4}\sum_{\substack{\text{all}\\ \text{spins}}} \left|\mathcal{M}(e^+ + e^- \to H^0 + Z^0)\right|^2 \;= \tag{4}$$

$$= \; A\left(\frac{e^2 M_Z}{s - M_Z^2}\right)^2 \times \left((p_{e^+}\cdot p_{e^-}) \;+\; \frac{2}{M_Z^2}(p_Z\cdot p_{e^+})(p_Z\cdot p_{e^-})\right)$$

where

$$A \;=\; \frac{1 + (1 - 4\sin^2\theta_W)^2}{(4\sin^2\theta_w)^2} \;\approx\; 1.15. \tag{5}$$

Note Lorentz invariance of the right hand side of eq. (4).

Your task is to calculate the total cross section for the process (1) as a function of the net energy $E_{\text{c.m.}}$ in the center-of-mass frame⋆ (see eq. (3) for the Lorentz-invariant formula) and the angular dependence of the partial cross section $d\sigma/d\Omega$ on the directions of the final Higgs and Z_0 particles.

⋆ The "center of mass" frame is the frame of zero net 3–momentum $\mathbf{P}_{\text{net}}$ of the initial (or final) particles. Strictly speaking, for relativistic particles this frame should be properly called the "center of energy frame", but everybody calls it the "center of mass frame" anyway.