1. First, a reading assignment: §6.1 and §6.4 of the *Peskin & Schroeder* textbook. Also, skim through §6.5 about multiple soft photons, real or virtual; never mind the techniques discussed in this section, but the results are important.

2. Now consider muon's gyromagnetic factor g. Experimentally, it has been measured with a very high precision (9 significant digits) and the theoretical calculations have a similarly high precision. At present, there is a very small discrepancy

$$g_\mu^{\rm exp} \; - \; g_\mu^{\rm theory} \; \approx \; (58 \pm 13^{\rm stat} \pm 12^{\rm syst}) \cdot 10^{-10}, \tag{1}$$

which is probably due new physics beyond the Minimal Standard Model (*i.e.*, loops involving some new particles, for example the superpartners). However, there is also some uncertainty in the photon-hadron coupling, which affects the g_μ at the two-loop order in QED. Usually, the photon-hadron coupling is obtained from the $e^+e^\rightarrow$hadrons cross-section, but it can also be derived from the vector spectral functions obtained from the $\tau \to \nu_\tau +$ hadrons decays, and the two method yield slightly different results: the former leads to eq. (1) while the later leads to a smaller discrepancy

$$g_\mu^{\rm exp} \; - \; g_\mu^{\rm theory} \; \approx \; (15 \pm 13^{\rm stat} \pm 12^{\rm syst}) \cdot 10^{-10}. \tag{2}$$

In this exercise we consider non-minimal versions of the Standard Model which contain some extra particles. Your task is to calculate the effect of such particles on the muon's magnetic moment at the one-loop level, and comparing your results to eq. (1) establish limits on masses and couplings of those extra particles.

(a) Let's extend the MSM by adding just one extra field Φ, a heavy neutral scalar of mass $M_S \simeq 200$ GeV which has a Yukawa coupling to the muon field Ψ,

$$\mathcal{L} \; \supset \; g\Phi \times \overline{\Psi}\Psi. \tag{3}$$

Calculate Φ's contribution to the muon's magnetic moment at the one-loop level. Then use your result to derive an upper limit on the Yukawa coupling g.

(b) A different extension of the Standard model contains an *axion*, a very light pseudoscalar field ϕ which couples to muons (and other leptons) according to

$$\mathcal{L} \supset \frac{\partial_\mu \phi}{f_a} \times \overline{\Psi}\gamma^5\gamma^\mu\Psi \approx \frac{2im_{\text{muon}}}{f_a}\,\phi \times \overline{\Psi}\gamma^5\Psi + \text{ a total derivative.} \tag{4}$$

The axion is a pseudo-Goldstone boson resulting from spontaneous breakdown of an axial symmetry at a very high energy scale $f_a \gg 100$ GeV; the symmetry is inexact but very good, so the axion's mass is non-zero but very small, $M_A \ll 1$ MeV. Calculate the axion's contribution to the muon's magnetic moment at the one-loop level. Then use your result to derive a lower limit on the axion scale f_a.

3. Finally, let's calculate the δ_2 counterterm of QED at the one-loop level and verify that it equals to the δ_1 counterterm we have calculated in class, including the finite parts of both counterterms.

The counterterms depends on the gauge (we shall address this issue in the next homework), so use the same gauge and regulators we have used in class: $D < 4$ dimensions to regulate the UV divergence, a tiny photon mass $m_\gamma^2 \ll m_e^2$ to regulate the IR divergence, and the Feynman gauge for the photon propagators, thus

$$= \frac{-ig^{\mu\nu}}{k^2 - m_\gamma^2 + i0}\,. \tag{5}$$

Start by calculating the $\Sigma^{1,\text{loop}}(\not{p})$ for the off-shell electron momenta p, then take the derivative $d\Sigma/d\not{p}$, and only then take the momentum on-shell, $\not{p} \to m_e$. Note that $\Sigma(\not{p})$ itself is infrared-finite, but its derivative has an IR singularity when the momentum goes on-shell, and that's why you need the IR regulator.

Note: You should get $\delta_2 = \delta_1$ *before you take the* $D \to 4$ *limit.* If this does not work, check your calculations for mistakes.