1. Back in 1996, Aharoni, Sonnenschein, Theisen, and Yankielowitz arXiv:hep-th/9611222 found a curious $SU(2) \times SU(2)$ SUSY gauge theory 'living' on a D3–brane probe located near intersection of two orientifold planes. In this problem, we study the renormalization group flow and the IR fixed points of this gauge theory from a purely 4D QFT point of view.

   Ignoring the 'stringy' degrees of freedom, we have $d = 4$, $\mathcal{N} = 1$ SUSY gauge theory with $G = SU(2) \times SU(2)$ and the following multiplets of chiral superfields:

$$A_1, A_2 \in (\mathbf{2}, \mathbf{2}), \qquad B_1, \ldots, B_8 \in (\mathbf{2}, \mathbf{1}), \qquad C_1, \ldots, C_8 \in (\mathbf{1}, \mathbf{2}) \tag{1}$$

   (gauge indices suppressed). The superpotential is

$$W \;=\; \lambda \sum_{i=1}^{4} B_i A_1 C_i \;+\; \lambda \sum_{i=5}^{8} B_i A_2 C_i \,. \tag{2}$$

   (a) List global symmetries of this model and show that it has only 3 independent anomalous dimensions: γ_A (same for A_1 and A_2), γ_B (same for all B_i), and γ_C (same for all C_i). Allow for different gauge couplings $g_1 \neq g_2$ of the two $SU(2)$ factors.

   (b) Calculate the exact β_λ, β_{g_1}, and β_{g_2} in terms of the anomalous dimensions and show that all three beta-functions vanish when $\gamma_B = \gamma_C = -\frac{1}{2}\gamma_A$. Argue that this leads to a line of fixed points in the (λ, g_1, g_2) coupling space.

   (c) Calculate the anomalous dimensions γ_A, γ_B, and γ_C to one-loop order and show that the fixed line lies at

$$g_1^2 \;=\; g_2^2 \;=\; \frac{16}{12}\,\lambda^2 \;+\; O(\lambda^4). \tag{3}$$

   (d) Show that this fixed line is IR-attractive. That is, if we start with some other couplings in the UV and let the RG run to lower energies, then in the IR limit the couplings will end somewhere on the fixed line (3).

(e) Any IR-attractive fixed point gives rise to an SCFT (super-conformal field theory), and a line (surface, *etc.*) of such fixed points makes a whole family of non-trivial SCFTs. Argue that for the model in question, this family of SCFTs includes both weakly-coupled and strongly-coupled theories.

2. In lieu of the second problem, a reading assignment: 1982 paper by Edward Witten, *"Constraints On Supersymmetry Breaking",* published in *Nuclear Physics* **B202** (1982), pages 252–316. This paper is not available on-line, but the PMA library has a hard copy.

   Update: Oscar Chacaltana have found a PDF file somewhere, and I have emailed it to all registered students on Thursday 10/8 about 5:15 PM. If any student didn't get a copy for some reason, please `email me` ASAP.