1. First of all, finish reading the Witten's paper I assigned last week.

2. Consider SQCD with $N = N_c - N_f \geq 2$. As explained in class, when quark masses are small (or zero) the squarks and antisquarks develop large VEVs and Higgs the $SU(N_c)$ gauge symmetry down to the $SU(N)$. The low-energy EFT is SYM coupled to the moduli $\mathcal{M}_{ff'} = \tilde{Q}_f Q'_f$ via moduli-dependence of the SYM's $\Lambda_I R$, and the gaugino condensation in the unbroken SYM generates the non-perturbative superpotential for the moduli,

$$W_{n.p.}(\mathcal{M}) \;=\; -N\left(\frac{\Lambda_{\rm SQCD}^{3N_c-N_f}}{\det(\mathcal{M})}\right)^{1/N}, \qquad W_{\rm net}(\mathcal{M}) \;=\; {\rm tr}(m\mathcal{M}) \;+\; W_{n.p.}(\mathcal{M}). \tag{1}$$

   (a) Suppose all quark flavors have non-zero (but small) masses, so the mass matrix m is invertible. Show that in this case, the SUSY vacuum equations

$$\frac{\partial W_{\rm net}}{\partial \mathcal{M}_{ff'}} \;=\; 0 \quad \forall f, f' \tag{2}$$

   have N_c distinct solutions

$$\mathcal{M}_{ff'} \;=\; -\left(m^{-1}\right)_{ff'} \times \left(\Lambda_{\rm SQCD}^{3N_c-N_f} \times \det(m)\right)^{1/N_c}, \qquad N_c \text{ branches.} \tag{3}$$

   These solutions correspond to N_c SUSY vacua of the massive SQCD, which has the same Witten index N_c as the quark-less SYM.

   (b) Now suppose there is only one flavor but it's massless. Your task is to calculate the scalar potential for the modulus $\Phi = (\mathcal{M} = \tilde{Q}Q)^{1/2}$ and show that for $\Phi \gg \Lambda_{\rm SQCD}$ it behaves like a *negative* power of $|\Phi|$. This potential does not have any minima at finite Φ, hence the theory does not have any stable vacua at all, supersymmetric or otherwise; instead, squark VEVs *run away to infinity.*

   Note: Classically, the flat direction Φ has $K(\Phi, \Phi^*) = 2\Phi^*\Phi$ and hence constant metric $g_{\Phi\bar{\Phi}} = 2$ (prove this!). In the quantum theory, this metric suffers from both perturbative and non-perturbative corrections, but as long as $\Phi \gg \Lambda$, the gauge coupling at the Higgs scale is weak and the corrections are small.

(c) Finally, consider several massless flavors. For simplicity, assume the moduli matrix is diagonal, $\mathcal{M} = \text{diag}(\phi_1^2, \phi_2^2, \ldots, \phi_{N_f}^2)$; classically, this corresponds to diagonal VEVs of the squark and antisquark matrices, $\left\langle Q_f^c \right\rangle = \phi_f \delta_f^c$, $\left\langle \tilde{Q}_c^f \right\rangle = \phi_f \delta_c^f$. Calculate the scalar potential for the ϕ_f (assuming all $\phi_f \gg \Lambda$) and show that they all run away to infinity — there are no stable vacua, supersymmetric or otherwise.