1. First, a reading assignment: Philip Argyres's 2001 lecture notes, sections §3.3–5, about SQCD with $N_f \geq N_c$, conformal and superconformal symmetry, and Seiberg duality.

2. Second, let's check 't Hoof's anomaly matching conditions for the Seiberg duality. Clearly, if two distinct gauge theories A and B have the same IR physics, they must have exactly the same un-broken flavor symmetries. Moreover, because the massless composite fermions are the same in both cases, their flavor anomalies must match those of the elementary fermions of either UV theory, hence the UV–UV anomaly matching conditions,

$$\begin{aligned} \mathrm{tr}_{(A)}\big(F\{F',F''\}\big) &= \mathrm{tr}_{(b)}\big(F\{F',F''\}\big) \quad \forall \text{ unbroken flavor charges } F,F',F'', \\ \text{and also } \mathrm{tr}_{(A)}(F) &= \mathrm{tr}_{(B)}(F) \quad \forall \text{ abelian } F, \end{aligned} \tag{1}$$

where $\mathrm{tr}_{(A)}$ means the trace over all elementary LH Weyl fermions of the UV theory A, and likewise $\mathrm{tr}_{(B)}$ is the trace over all elementary LH Weyl fermions of the UV theory B.

Your task is to verify these anomaly-matching conditions for a specific pair of Seiberg-dual theories. Theory A is pure SQCD with N_c^A colors and $N_f \geq N_c^A + 2$ flavors. It has quarks $Q^{c,f}$, antiquarks $\widetilde{Q}_c^f$, and the gauge fields V^a; there is nothing else. Theory B is also SQCD-like but with extra singlets. Specifically, it has the same number N_f of flavors as (A) but a different number of colors, $N_c^B = N_F - N_c^A$. Besides the gauge fields v^a, it has quarks q_f^c, antiquarks $\tilde{q}_{c,f}$, and N_f^2 gauge-singlet fields $M^{ff'}$ with Yukawa couplings to the quarks and antiquarks,

$$W_{\text{tree}}^B = \lambda \sum_{c,f,f'} q_f^c M^{ff'} \tilde{q}_{c,f} \,. \tag{2}$$

Note that the quarks and antiquarks have upper flavors indices in theory A but lower flavor indices in theory B. This denotes their opposite quantum numbers with respect to the $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry group: The A-quarks $Q^{c,f}$ form the $\mathbf{N_f}$ multiplet of $SU(N_f)_L$ while the B-quarks q_f^c form the conjugate $\overline{\mathbf{N_f}}$ multiplet. Likewise, the A-antiquarks $\widetilde{Q}_f^c$ form the $\mathbf{N_f}$ of the $SU(N_f)_R$ while the B-antiquarks $\tilde{q}_{c,f}$ form the $\overline{\mathbf{N_f}}$.

(a) Tabulate the $SU(N_c) \times SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$ quantum numbers of all the elementary fermions of each theory. (The R–symmetry here is the color-anomaly-free combination of the pure-R and the axial symmetry.) Also, check that the theory B does not have any other anomaly-free symmetries.

(b) Calculate all the flavor anomalies of each theory and check that they match.

3. Finally, an exercise in conformal symmetry and the radial quantization. Suppose a CFT (supersymmetric or otherwise) has an abelian gauge symmetry and let $|F^{\mu\nu}\rangle$ be the primary state (in the Hilbert space on the S^3) corresponding to the $F^{\mu\nu}$ operator at $x = 0$. Consider the descendant states

$$|J^\nu_{\rm el}\rangle \;=\; P^\mu\,|F^{\mu\nu}\rangle \quad \text{and} \quad \left|J^\nu_{\rm mag}\right\rangle \;=\; P^\mu\left|\tilde{F}^{\mu\nu}\right\rangle \;=\; \tfrac{1}{2}\epsilon^{\mu\nu\kappa\lambda}P_\mu\left|F^{\kappa\lambda}\right\rangle. \tag{3}$$

Use $P^\dagger_\mu = -iK_\mu$ (in radial quantization) and the conformal algebra to calculate the Hilbert norms $\langle J^\nu_{\rm el}|J^\nu_{\rm el}\rangle$ and $\langle J^\nu_{\rm mag}|J^\nu_{\rm mag}\rangle$ of these states in terms of the scaling dimension Δ of the $F^{\mu\nu}$ field. Show that both norms are zero if $\Delta = 2$ and both are positive if $\Delta > 2$.

Use this result to argue that either the abelian gauge field is free (in the IR limit where the theory is conformal) or else it must couple to both electric and magnetic charges.