General Relativity

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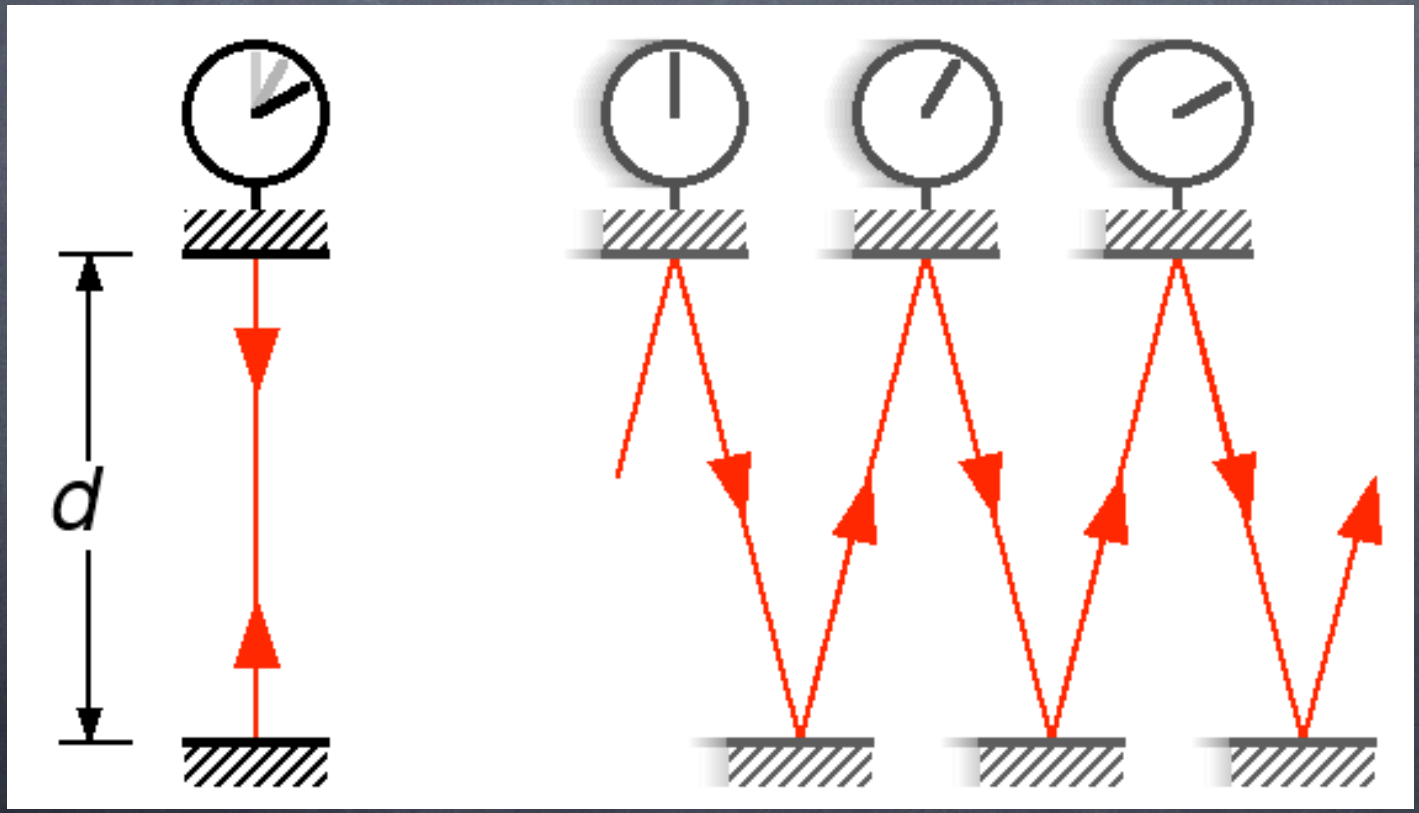
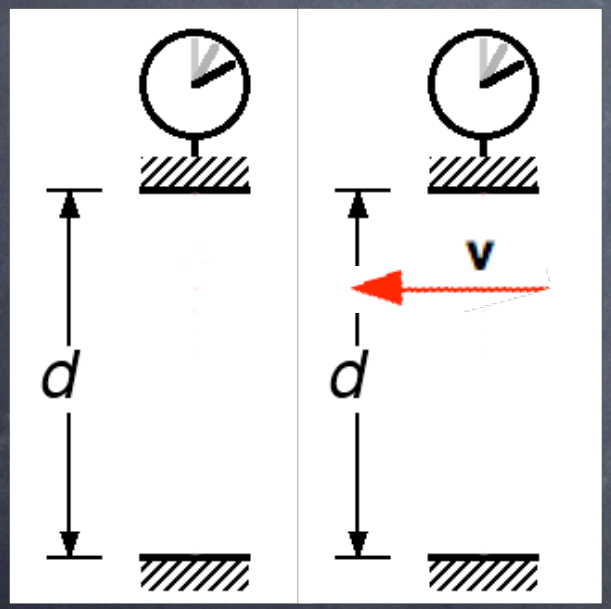
Maxwell: EM waves with velocity c= 299792458m/sec

- Einstein: c with respect to what?
  - Every observer measures the same speed!
- All observers see the same laws of physics!

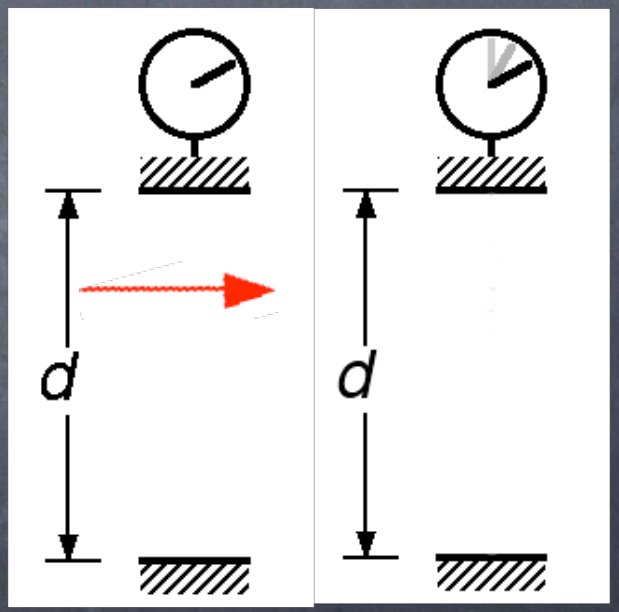
"Light Clock"



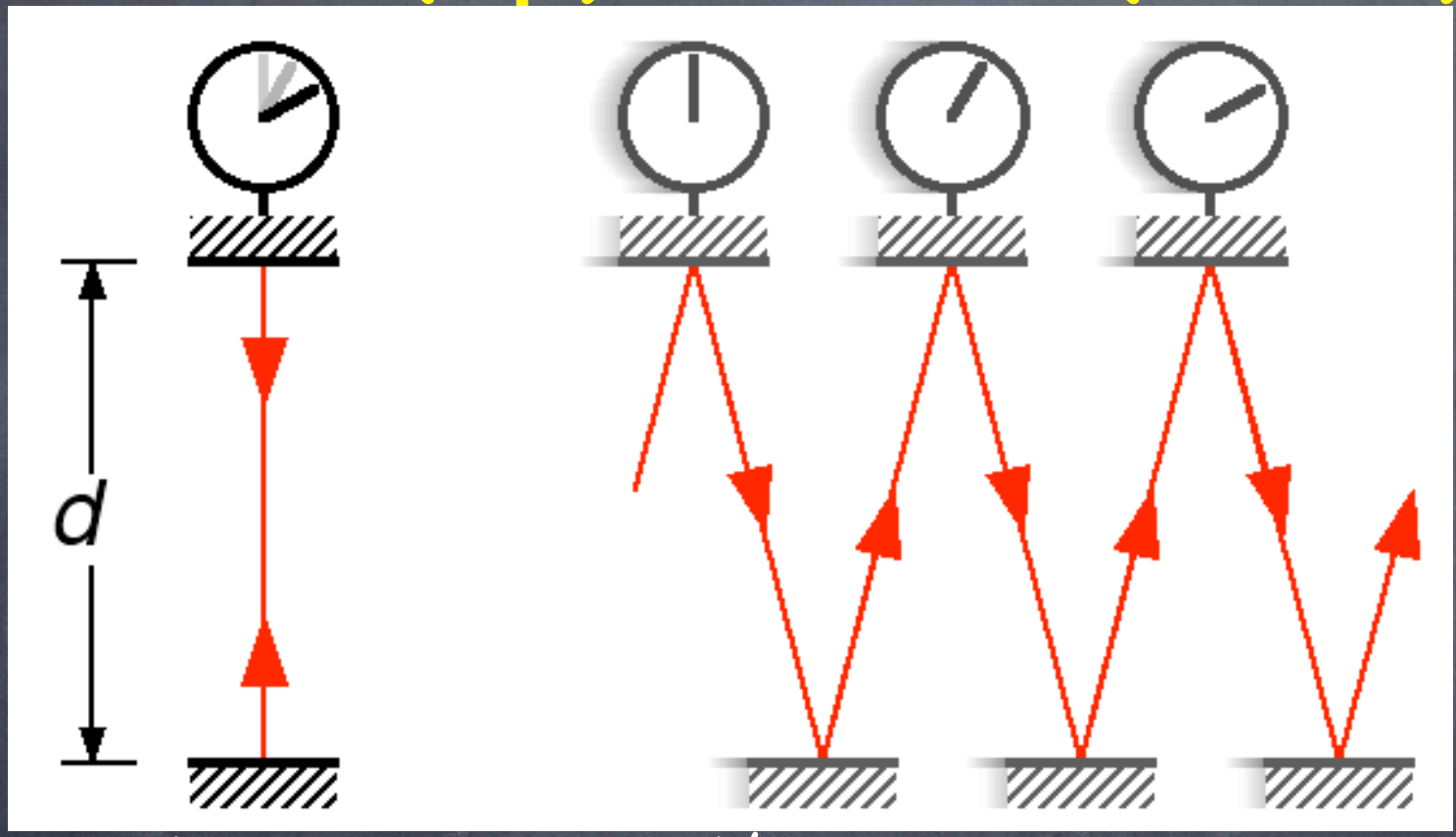
Symmetry Requires "d" Is Same For Both clocks



- these situations depend on who is observing



Tick (up) - Tock (down)



- Tick: time at rest = d/c
- Tick: time seen from nonmoving frame for tick on moving clock $\Delta t_{MG} = \sqrt{[d^2 + (v\,\Delta t_{MG})^2]}/c$ > rest clock time.

Result: $\Delta t_{MG} = \Delta t_{MM}/\sqrt{[1 - (v/c)^2]}$

- Time dilation is real
- applies to all clocks (including biological)
  - twin paradox

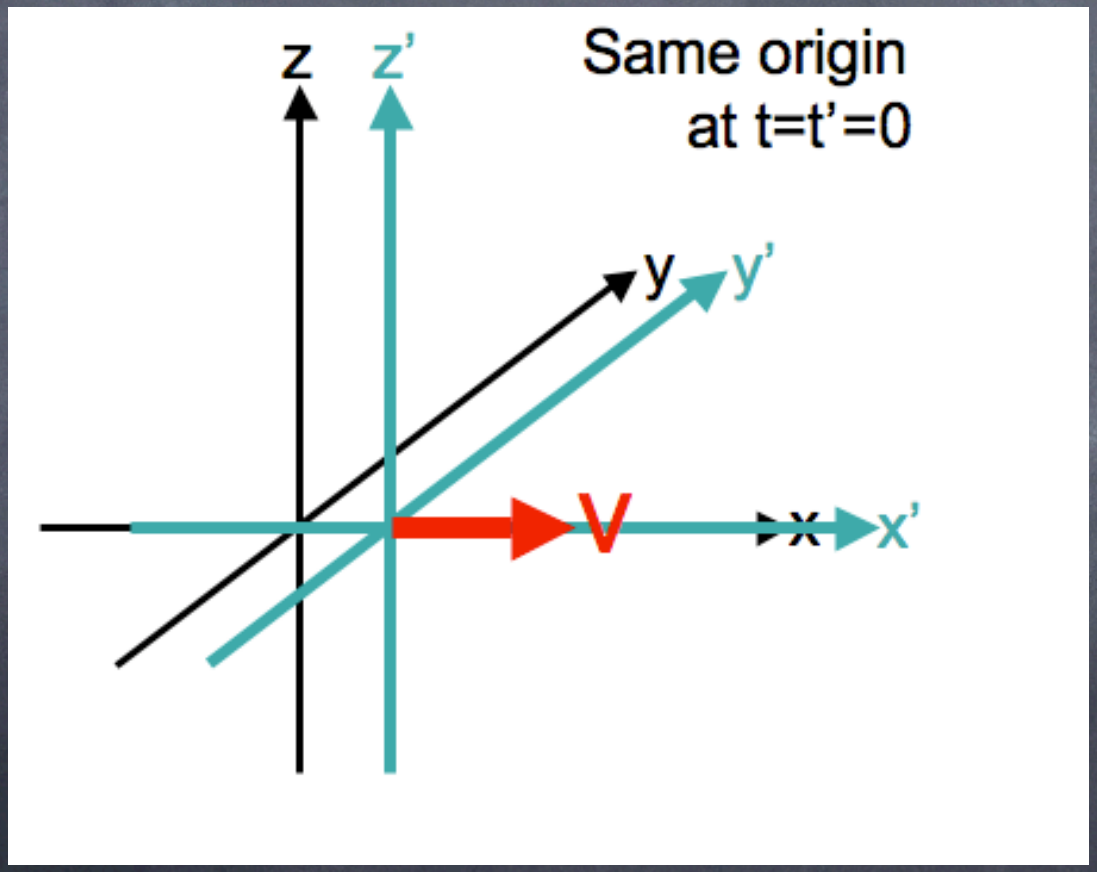
Length Contraction:

$$\text{length}_{MG} = \sqrt{[1 - (v/c)^2]}\ \text{length}_{MM}$$

Lorentz Transformations

By looking for a transformation from one frame to another, with the requirement that the Maxwell equations have identically the same forms, Einstein found the following way to relate measurements by an observer in one coordinate frame to measurements by an observer in another frame ---

Two Observers with Personal Reference "Frames"



Galilean Transformations

$$x' = [x-(v/c)ct]$$

$$y' = y$$

$$z' = z$$

$$ct' = [ct]$$

Lorentz Transformations

$$x' = \gamma[x-(v/c)ct]$$

$$y' = y$$

$$z' = z$$

$$ct' = \gamma[ct-(v/c)x]$$

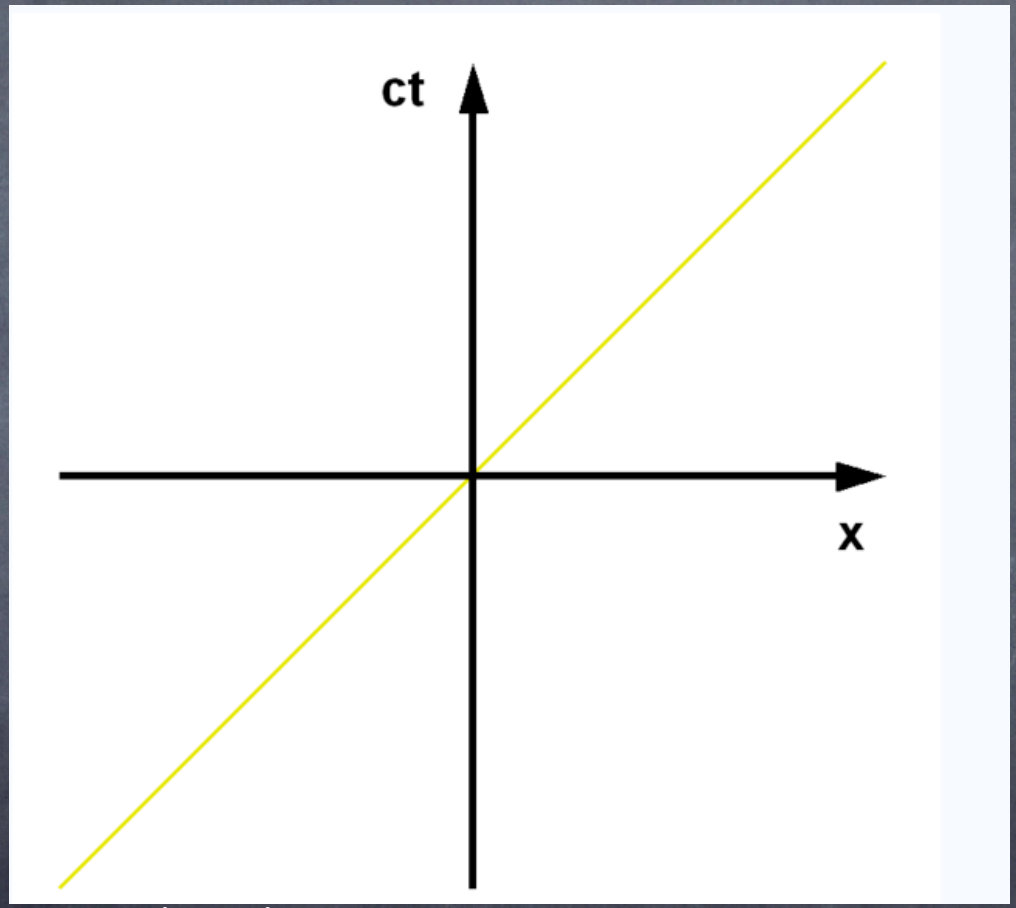
where

$$\gamma = 1/\sqrt{[1-(v/c)^2]}$$

(Lorentz had discovered these transformations in a physical model of the electron in an EM field.)

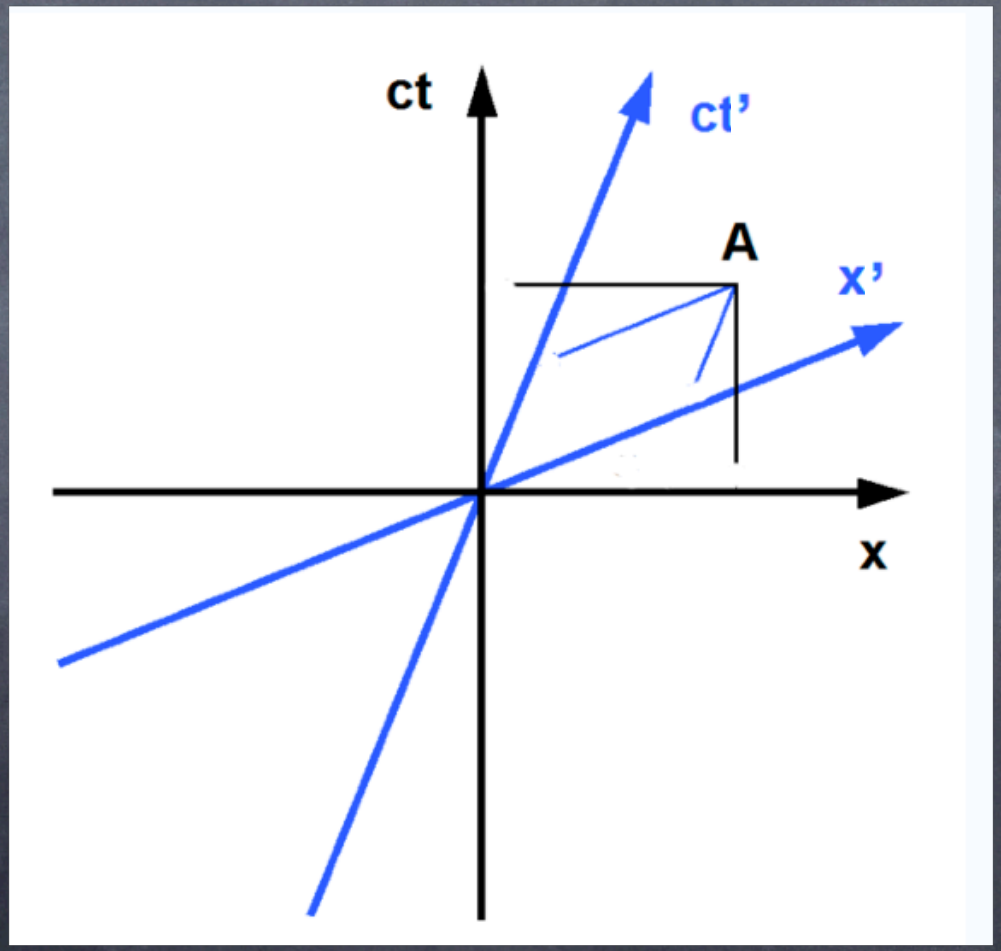
Lorentz transformations imply time dilation and length contraction, but have dramatic new implications for causality and "spacetime" (rather than "space" and "time").

Spacetime Diagram



Using "ct" makes light path 45°

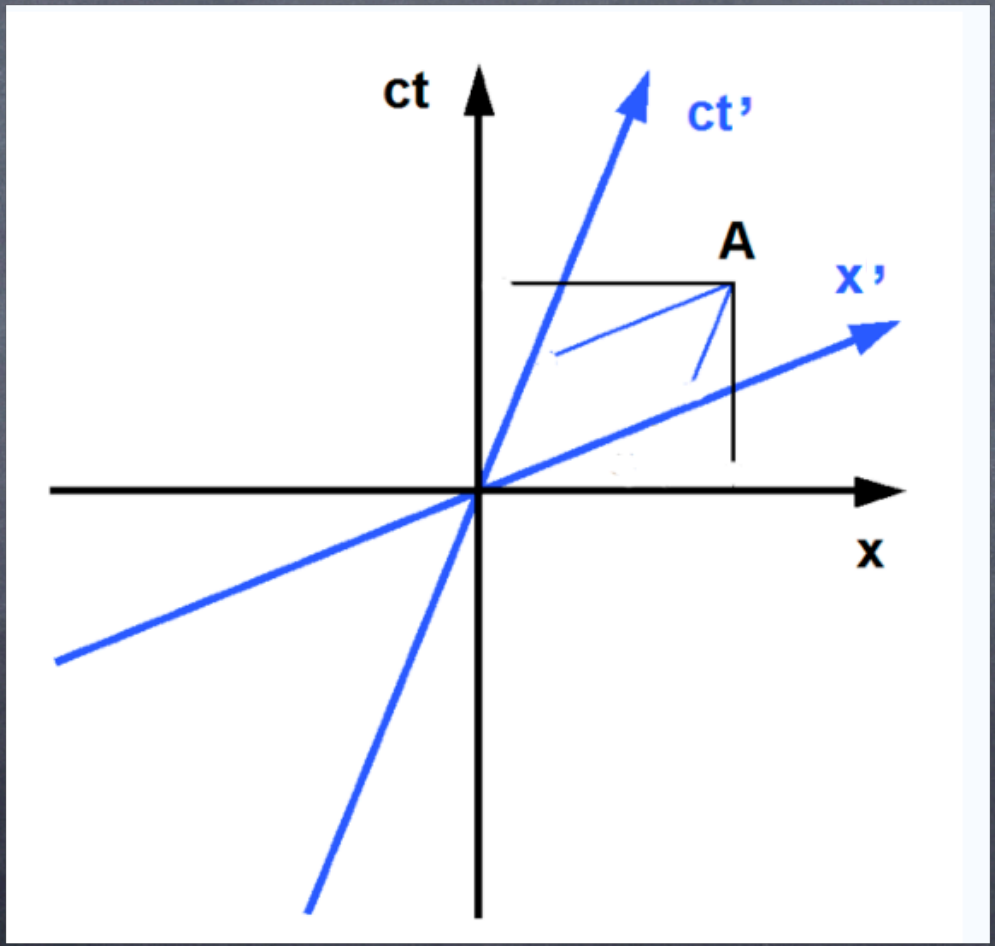
Two "observers", one moving



Lorentz transformations merge space and time into Spacetime!

$$x' = \gamma[x-(v/c)ct]$$
$$y' = y$$
$$z' = z$$
$$ct' = \gamma[ct-(v/c)x]$$

Simultaneity is Relative!



Observational status: twin paradox

Science 14 July 1972:
Vol. 177. no. 4044, pp. 168 – 170
DOI: 10.1126/science.177.4044.168

ARTICLES

Around-the-World Atomic Clocks: Observed Relativistic Time Gains

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"These results provide an unambiguous empirical resolution of the famous clock "paradox" with macroscopic clocks."

Time dilation: muons

- rest half life 1.6 x 10^{-6} sec
- produced at 10 km height, velocity v = 0.98c

10km/(3x10^5 km/sec) ≈ 3.3x10^{-5} sec ≈ 20 half lives

2^{-20} ≈ 0.3x10^{-6} : practically no muons should reach the earth. But they are detected!

The reason: Time dilation

Time dilation: muons

$v = 0.98 \Rightarrow \gamma \approx 5$

- muon clock running 5 times slower!
- $\Delta t_{MG} = \gamma \Delta t_{MM}$

- trip takes muon only 20/5 = 4 half lives

$2^{-4} \approx 0.06$: 1/16 of muons reach the earth.

No wonder they are detected!

Length contraction: muons

$v = 0.98 \Rightarrow \gamma \approx 5$

The muon sees itself at rest, earth speeding upward at 0.98c. Thus the muon - earth distance at the start is contracted :

$$D\sqrt{[1-(v/c)^2]}$$

$$\text{Distance} = 10\text{km} \times \sqrt{[1-(v/c)^2]}$$

$$= 10 \text{ km}/\gamma = 2 \text{ km}$$

Length contraction: muons

$v = 0.98 \Rightarrow \gamma \approx 5$

$2\text{km}/(3\text{x}10^5\ \text{km/sec}) \approx 6.6\text{x}10^{-6}\ \text{sec} \approx 4$ half lives

Same result as computed in Earth frame!

$2^{-4} \approx 0.06$: 1/16 of muons reach the earth.

Other well known implications

(derivations omitted)

$$E = mc^2$$

Energy and mass are interchangeable!

$$m = \gamma m_0 = m_0 / \sqrt{[1-(v/c)^2]}$$

Mass of a moving object increases

$$E = m_0c^2 + KE$$

Energy equivalent of mass increase is Kinetic Energy

General Relativity

Einstein's Description of Gravity

- fully **Geometric** formulation
- replaces Newton's gravitational **Force**

Basic motivation:

Equivalence Principle

Newtonian Equivalence Principle

Everything falls under gravity with the same acceleration!

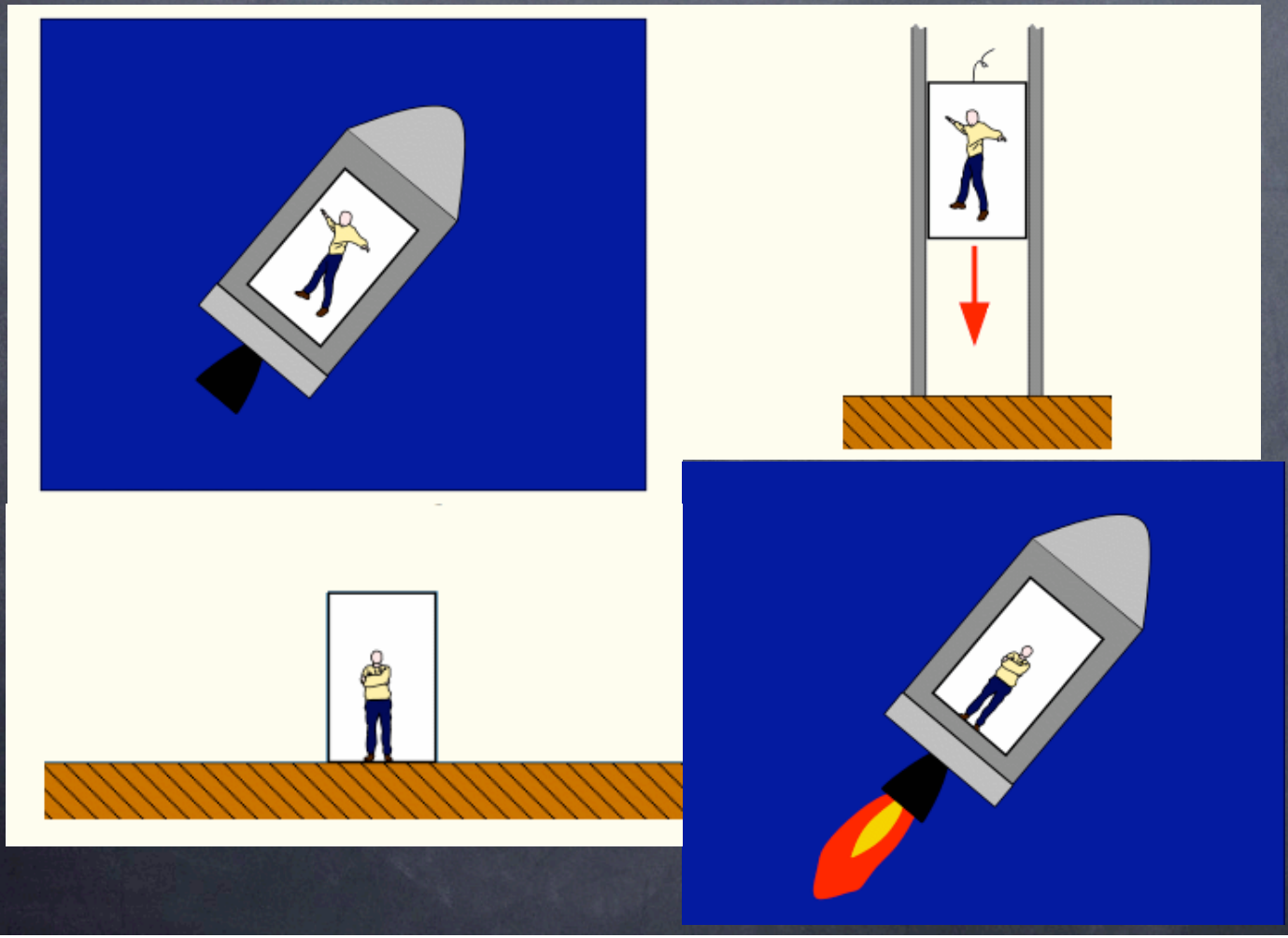
$$m_i \vec{g} = -\frac{GM_\oplus m_g}{r^3}\vec{r}$$

inertial mass gravitational mass

inertial mass = (passive) gravitational mass

Different from E&M

Practical effect: Einstein Elevator



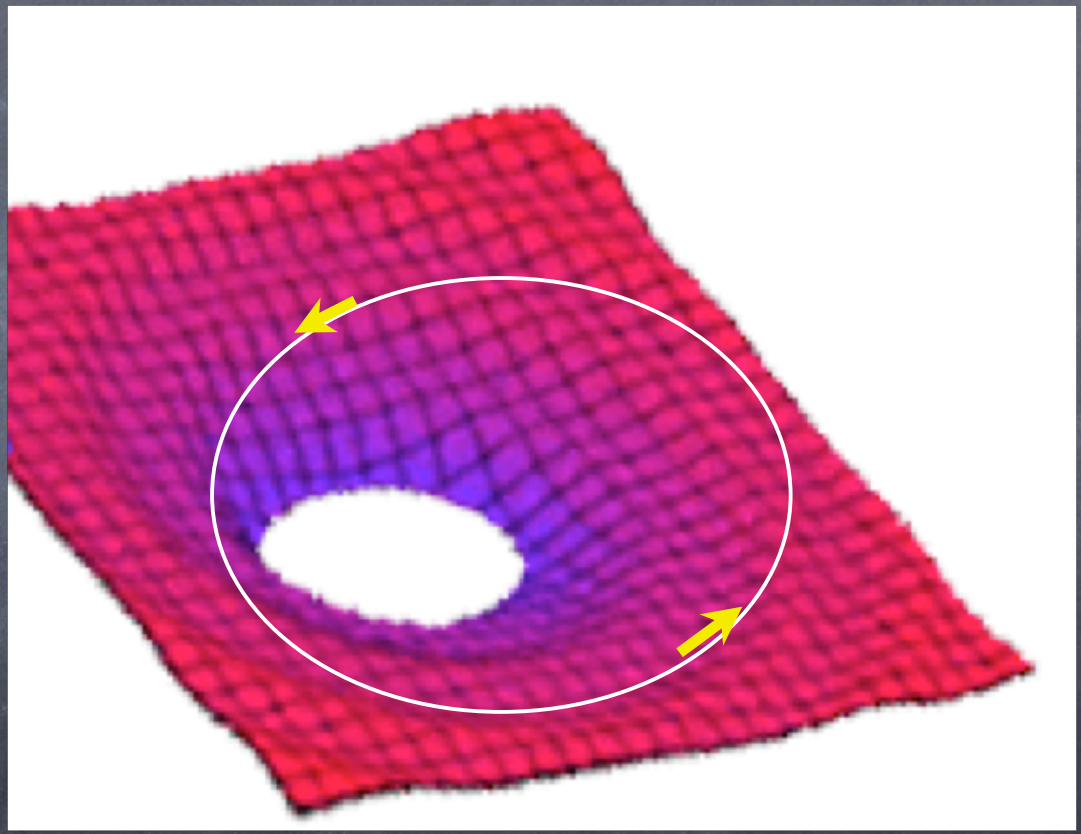
Why is this true? (Theoretical Question)

Newton: because $m_i = m_g$

Einstein: Because objects are following the same path in curved spacetime

Gravity is Curvature!

Curvature determines orbits



Newtonian orbits follow from Relativistic calculations

- Einstein equation computes curvature from matter source (e.g. sun)
- curvature determines path of particle (e.g. Earth)

(another equation)

For Weak Fields GR Equations very similar to Newtonian

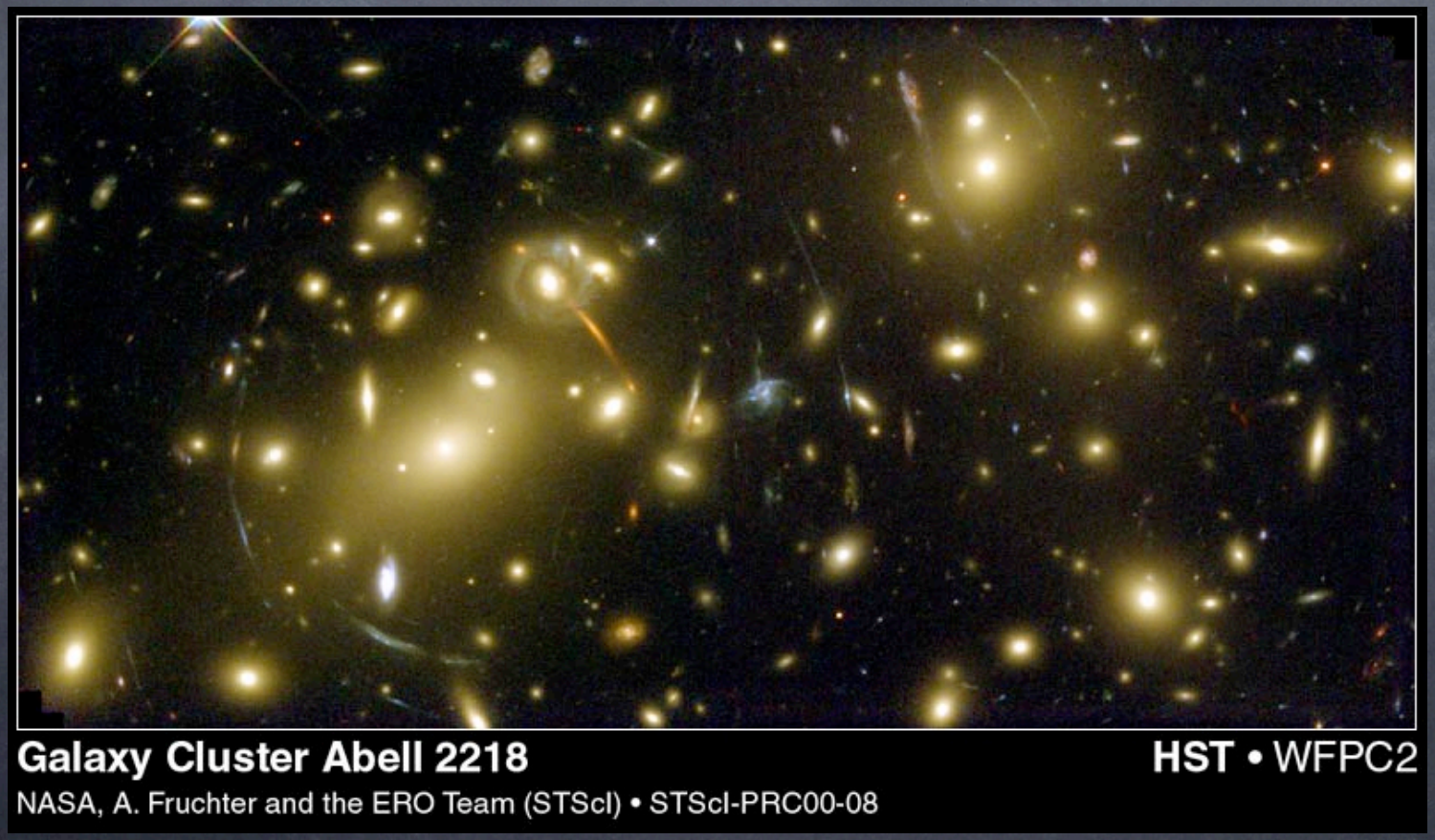
- Einstein equation very similar to Newtonian definition of Gravitational field
- Motion of object (e.g. planet around the Sun) very similar to Newtonian motion

Subtle GR Effects in Solar System

Solar gravitational field: 10^{-6}

- Small precession of perihelion of planets ≈ 42″ per century
- Small deflection of light ≈ 10^{-6} radians ≈ 1″.75

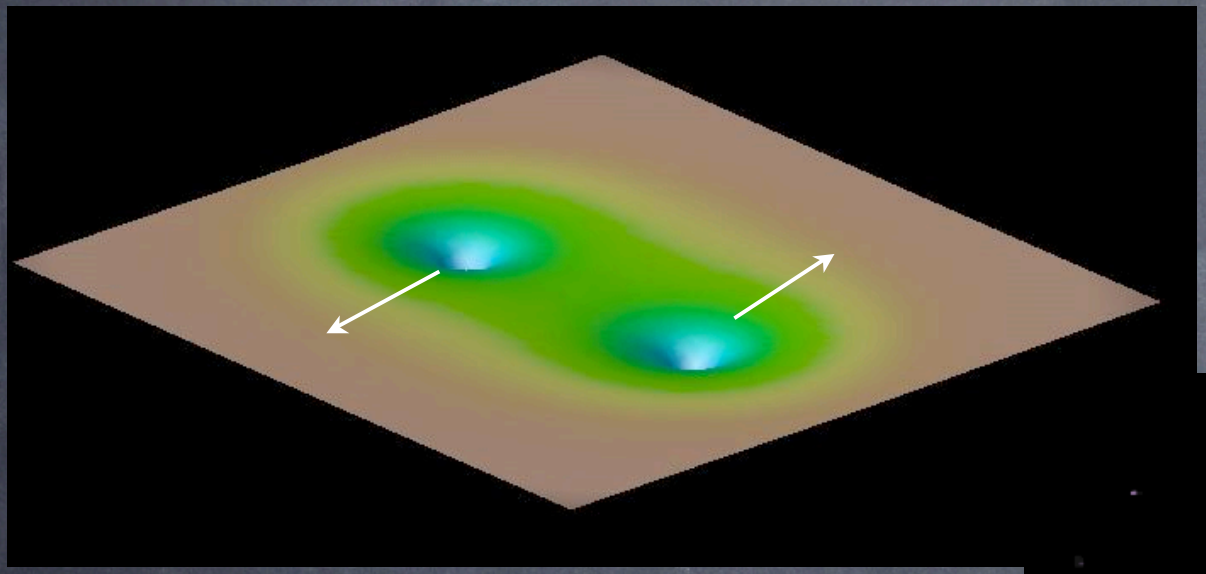
Strong/Big Fields Produce Big Effects



Galaxy Cluster Abell 2218 **HST** • WFPC2

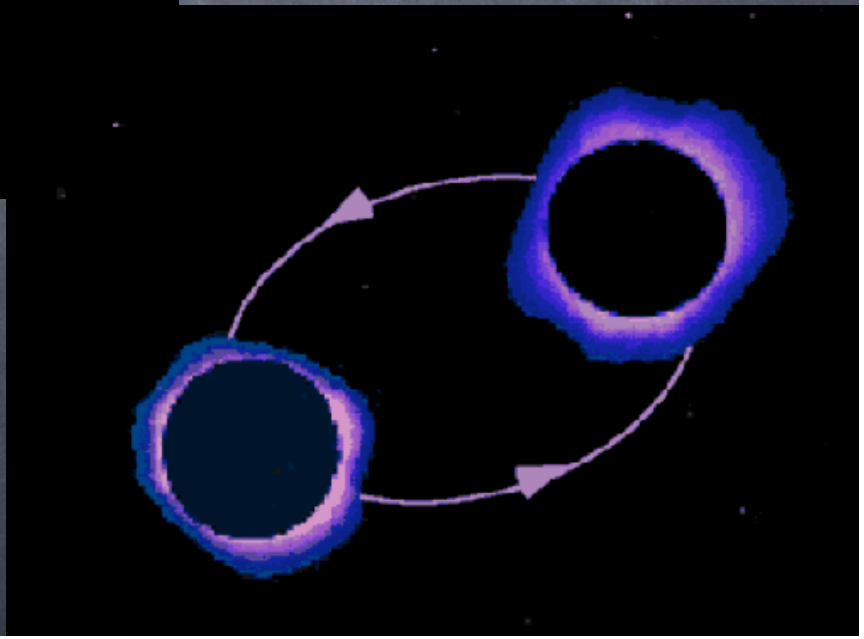
NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

Strong Fields: Binary Black Holes in Orbit

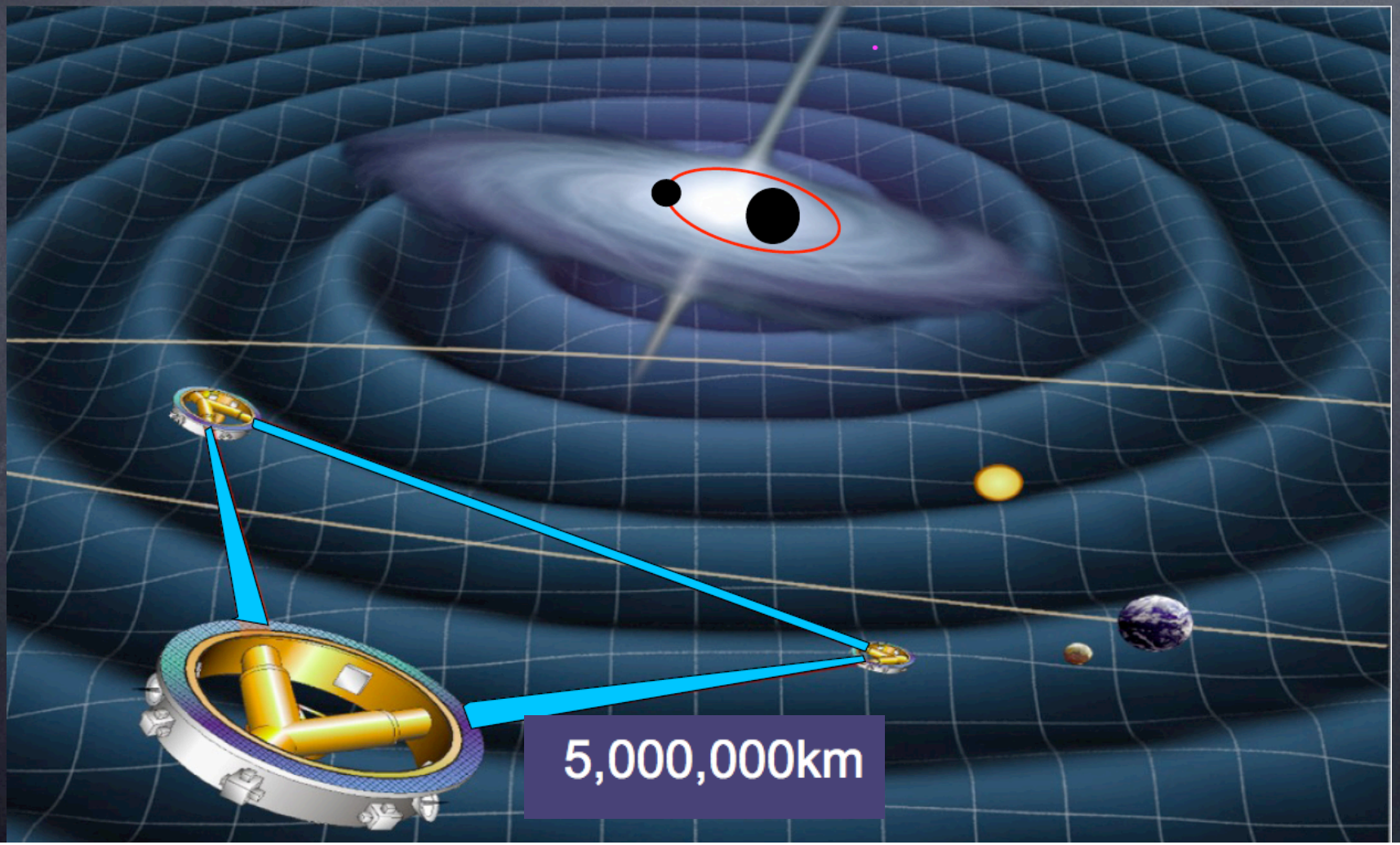


Orbiting strong curvature

Strong Astrophysical source of Gravitational waves



Gravitational Waves and Detector



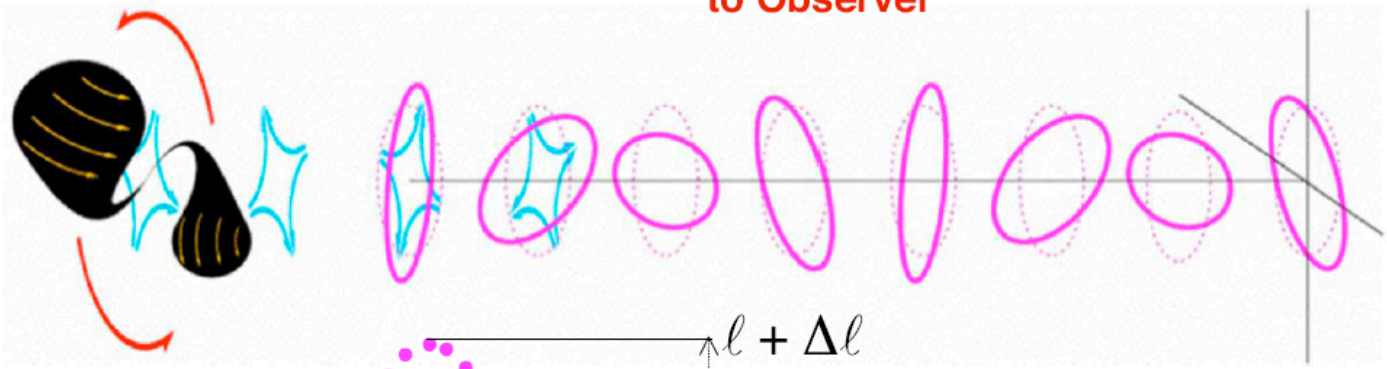
Strong Field Production of Gravitational Waves

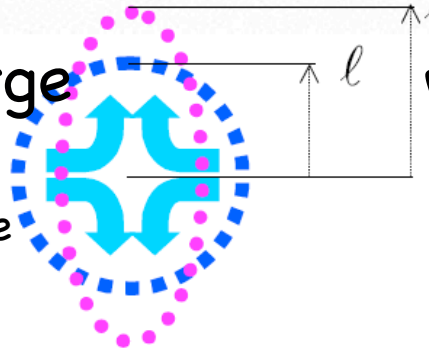
Gravitational Radiation

Source: Bulk Motion Produces Changing Tidal Field

Oscillating Tidal Field Propagates (Unobstructed) to Observer

Observer Detects Distortion Strain





$\frac{\Delta\ell}{\ell}$ very large

≈ 1

at source

waves are transverse

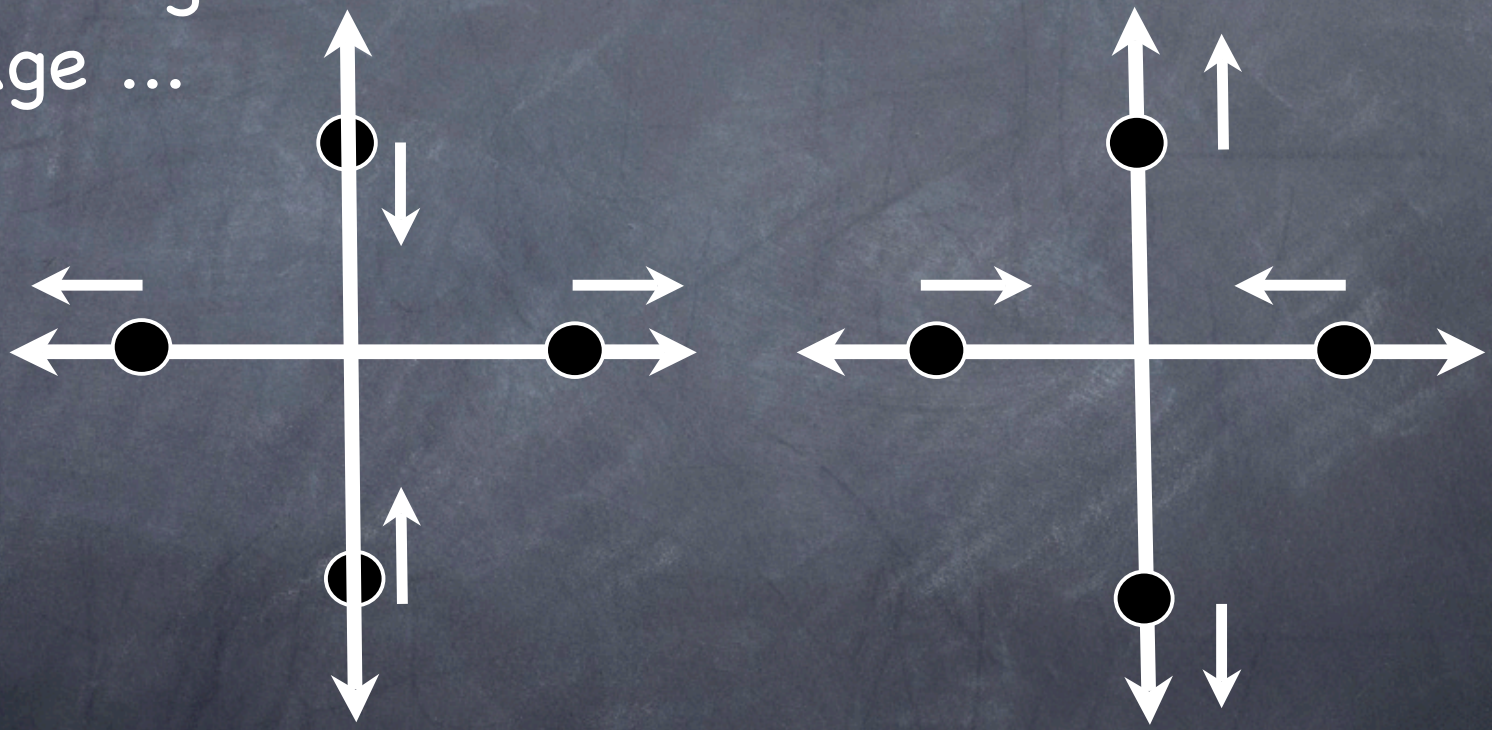
$\frac{\Delta\ell}{\ell}$ very small

$\approx 10^{-22}$

when detected

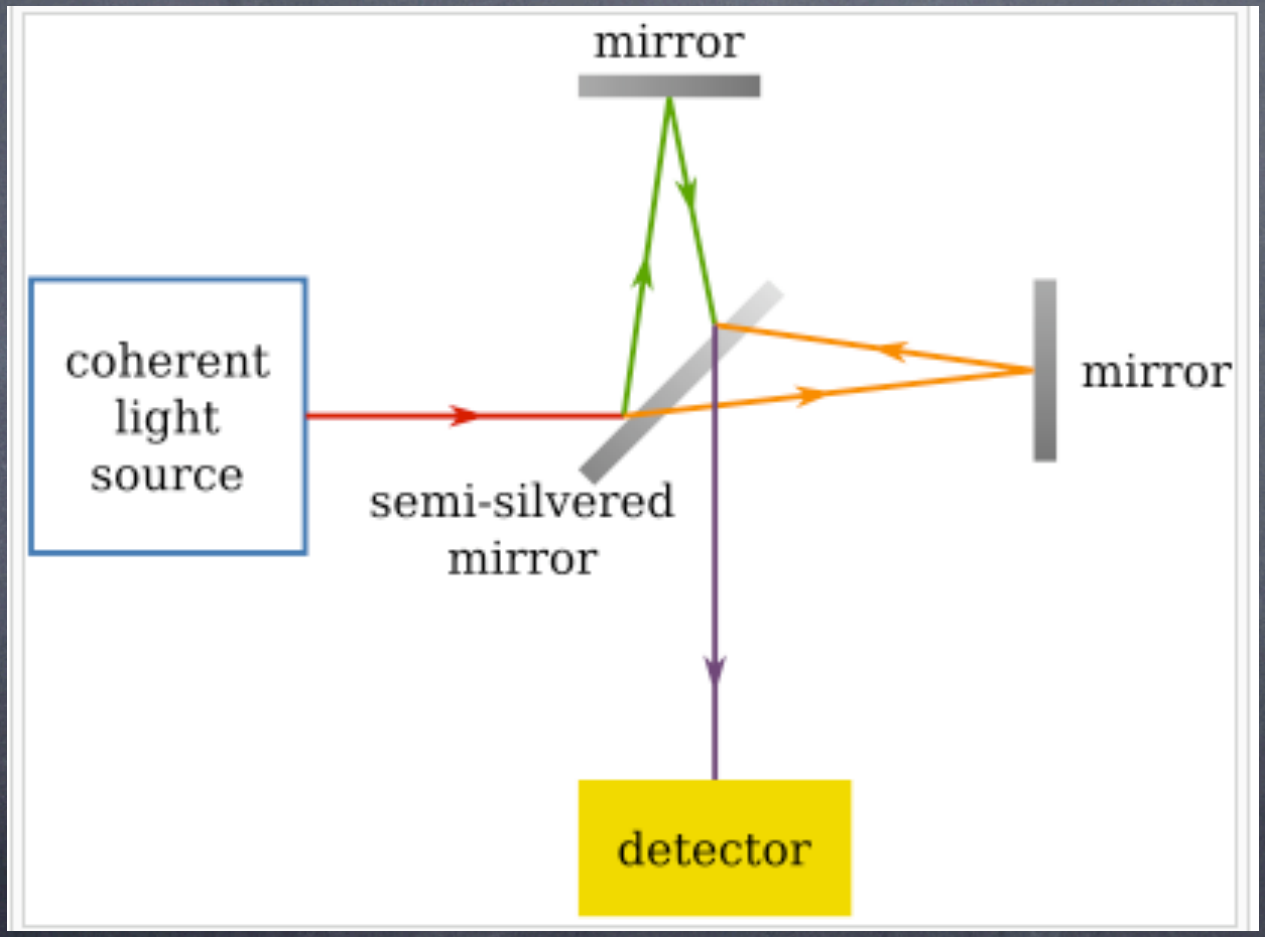
What Gravitational Radiation Does

Wave moving into page ...



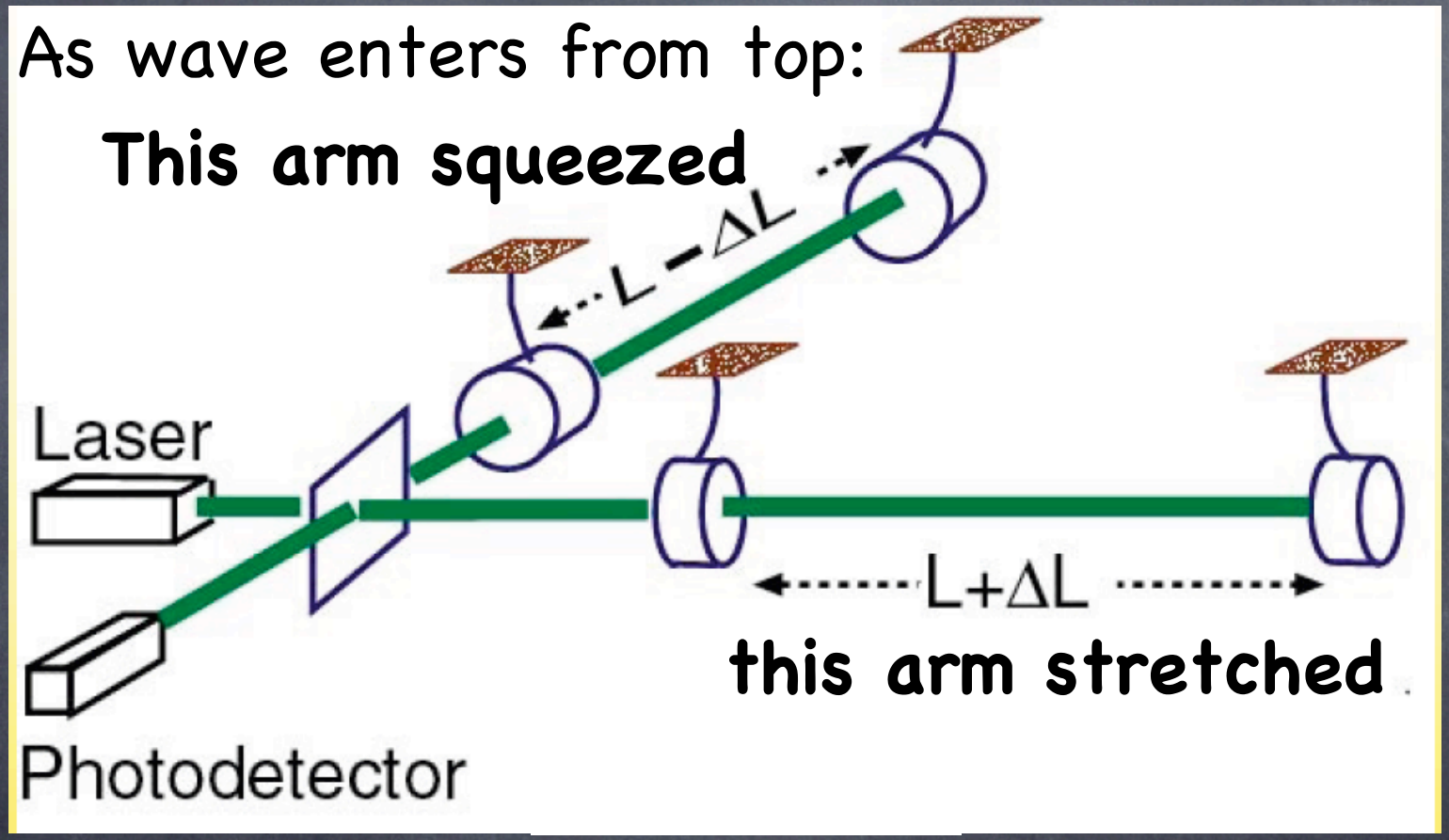
produces oscillatory transverse distortion. One transverse direction lengthens while the other shrinks. Then reverses.

Interferometer- Arm length difference changes light at detector



path difference → phase difference → interference

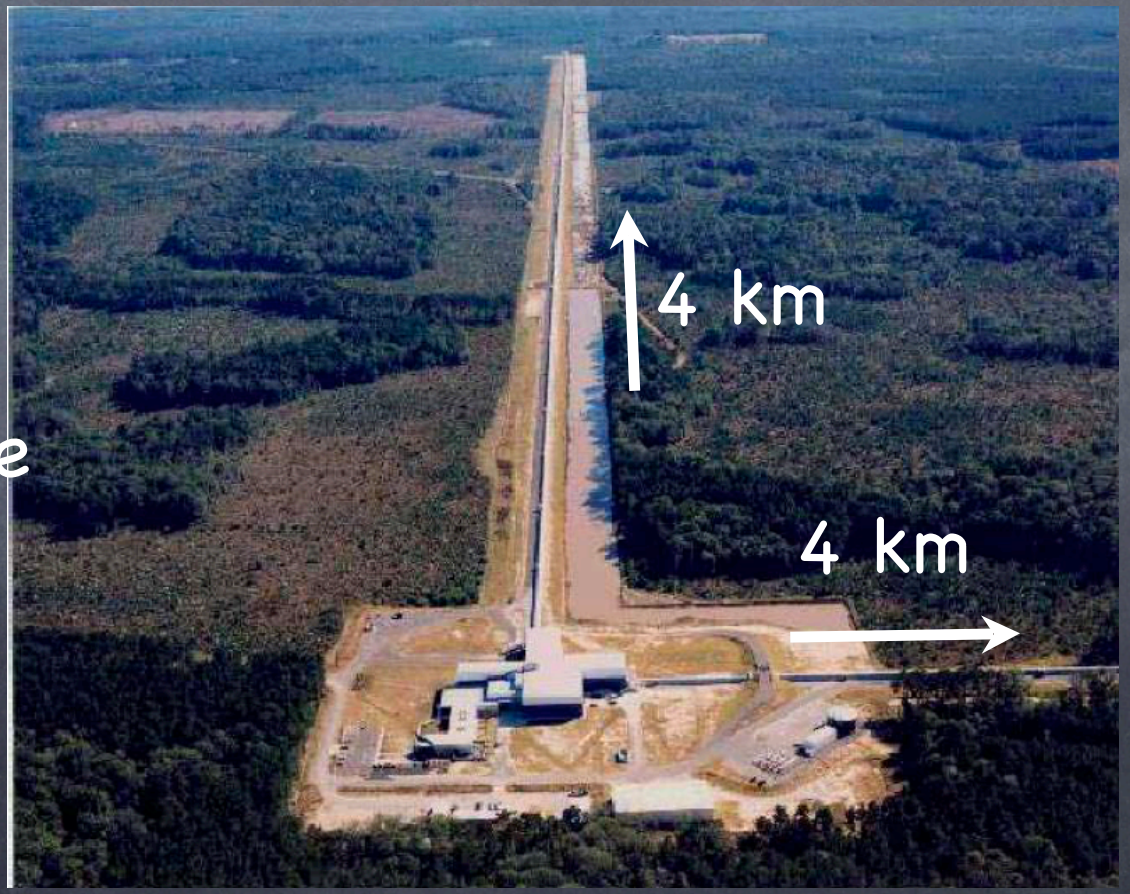
Interferometer Detector (LIGO)



Oscillating difference in lengths causes phase difference in arms, interference changes light intensity at the photodetector

LIGO: Interferometer detector

LIGO has sites in Lousisiana (4km detector) and in Washinton state (4km and 2km detectors)



Louisiana Site

No detections so far,
but...

LIGO is being upgraded to examine 1000 x the volume!