Problem #**1:**

There are two forces acting on the top bead: Its own weight $m_t g$ and the Coulomb force from the bottom bead,

$$F_C = \frac{kq_tq_b}{r^2}. \tag{1}$$

In equilibrium, the two forces cancel each other, which means that they have opposite directions and equal magnitudes

$$|F_C| = \frac{k|q_t|\,|q_b|}{r^2} = m_t g. \tag{2}$$

Solving this equation for the magnitude of the bottom bead's charge, we find

$$|q_b| = \frac{m_t g \times r^2}{k|q_t|} = \frac{(0.020\text{ kg}) \times (9.8\text{ N/kg}) \times (0.30\text{ m})^2}{(9.0 \cdot 10^9\text{ N m}^2/\text{C}^2) \times (0.50 \cdot 10^{-6}\text{ C})} \approx 3.9 \cdot 10^{-6}\text{ C}. \tag{3}$$

The sign of the bottom bead's charge follows from the direction of the electrostatic force between the two beads. Equilibrium of the top bead requires the force from the bottom bead to be directed vertically up — otherwise it would not cancel the gravity force — so the two beads should repel each other rather than attract. Since charges of like sign repel while charges of opposite signs attract, it follows that the bottom bead's charge should have the same sign as the top bead's — which happens to be positive.

Altogether, equilibrium of the top bead requires the bottom bead's charge to be $q_b = +3.9\ \mu$C.

Problem #**2:**

(a) Perfect efficiency of the motor means that the mechanical power it produces is equal to the electric power it consumes,

$$P_{\text{mechanical}} = P_{\text{electric}} = V \times I. \tag{4}$$

The second equality here is the formula for the electric power. It applies to both direct and alternating currents, provided that for the AC V is the RMS (root(mean square)) voltage and I is the RMS current.

For the motor in question we know the power $P = 8$ hp $\approx 6,000$ W and the RMS voltage of the AC power supply. Consequently, the RMS current through the motor is

$$I_{\rm rms} \;=\; \frac{P}{V_{\rm rms}} \;=\; \frac{6,000\ {\rm W}}{240\ {\rm V}} \;=\; 25\ {\rm A}. \tag{5}$$

(b) Instantaneous voltage on a motor and the instantaneous current through it are related by the motor's electric resistance R,

$$V(t) \;=\; R \times I(t). \tag{6}$$

The RMS averages of the voltage and the current are also related by the same resistance,

$$V_{\rm rms} \;=\; R \times I_{\rm rms} \tag{7}$$

so we can apply the Ohm's law to the AC current and voltage in the same way we would apply it to DC.

The AC voltage on the motor in question is $V_{\rm rms} = 240$ V while the current through it is $I_{\rm rms} = 25$ A (see part (a)). Hence the electric resistance of the motor is

$$R \;=\; \frac{V_{\rm rms}}{I_{\rm rms}} \;=\; \frac{240\ {\rm V}}{25\ {\rm A}} \;=\; 9.6\ \Omega. \tag{8}$$

PS: If you are interested in AC electric circuits beyond this *elementary* physics class, you should know that the formula $P = V \times I$ for the AC electric power works only when the current and the voltage have the same phase,

$$\begin{aligned} I(t) \;&=\; \sqrt{2} I_{\rm rms} \times \sin(2\pi f t + \phi_i), \\ V(t) \;&=\; \sqrt{2} V_{\rm rms} \times \sin(2\pi f t + \phi_v), \\ \text{for}\quad \phi_i \;&=\; \phi_v\,. \end{aligned} \tag{9}$$

For different phases $\phi_i \neq \phi_v$ there is a more complicated formula

$$P_{\rm avg} \;=\; V_{\rm rms} \times I_{\rm rms} \times \cos(\Delta\phi = \phi_i - \phi_v). \tag{10}$$

For simple circuits involving light bulbs, resistors, *etc.*, the voltage and the current usually have same phases, but for circuits involving inductance coils or capacitors there usually is some phase difference $\Delta\phi = \phi_i - \phi_v \neq 0$.

The Ohm's law for capacitors or inductance coils is also tricky. It does not work for the *instantaneous* currents and voltages,

$$V(t) \neq R \times I(t) \quad \text{for capacitors or inductance coils.} \tag{11}$$

However, there is an Ohm-like formula for the RMS averages,

$$V_{\mathrm{rms}} = Z \times I_{\mathrm{rms}} \tag{12}$$

where Z is called the *impedance*. Unlike the ordinary electric resistance R, the impedance Z depends on the frequency of the alternating current.

For the purpose of this test, I have assumed that the current through the motor has the same phase as the voltage on it, thus $\Delta\phi = 0$. For such a motor, the electric power is simply $P = V_{\mathrm{rms}} \times I_{\mathrm{rms}}$ and we may ignore the distinction between the electric resistance and the impedance.

Problem **#3:**

(a) In this series circuit, the same current flows through all the resistors while the voltages add up. Consequently, the resistances of the three resistors add up,

$$R_{\mathrm{net}} = R_1 + R_2 + R_3 = 30\ \Omega + 30\ \Omega + 30\ \Omega = 90\ \Omega. \tag{13}$$

(b) In this parallel circuit, the same voltage applies to all the resistors while the currents through them add up. Consequently, the *inverse resistances* of the three resistors add up,

$$\frac{1}{R_{\mathrm{net}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{30\ \Omega} + \frac{1}{30\ \Omega} + \frac{1}{30\ \Omega} = \frac{3}{30\ \Omega} = \frac{1}{10\ \Omega}, \tag{14}$$

so $R_{\mathrm{net}} = 10\ \Omega$.

(c) This circuit has a series sub-circuit of two resistors, and the whole circuit is a parallel connection of that sub-circuit and another resistor. For such 'nested' circuit,

$$R_{\rm sub} = R_1 + R_2 \quad \text{and} \quad \frac{1}{R_{\rm net}} = \frac{1}{R_{\rm sub}} + \frac{1}{R_3}\,. \tag{15}$$

Numerically,

$$\begin{aligned} R_{\rm sub} &= 30\ \Omega + 30\ \Omega = 60\ \Omega, \\ \frac{1}{R_{\rm net}} &= \frac{1}{60\ \Omega} + \frac{1}{30\ \Omega} = \frac{1}{20\ \Omega}\,, \end{aligned} \tag{16}$$

thus $R_{\rm net} = 20\ \Omega$.

(d) Finally, a different kind of nested circuit: It has a parallel sub-circuit of two resistors, and the whole circuit is a series connection of that sub-circuit and another resistor. For this circuit,

$$\frac{1}{R_{\rm sub}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{and} \quad \frac{1}{R_{\rm net}} = R_{\rm sub} + R_3\,. \tag{17}$$

Numerically,

$$\begin{aligned} \frac{1}{R_{\rm sub}} &= \frac{1}{30\ \Omega} + \frac{1}{30\ \Omega} = \frac{1}{15\ \Omega}\,, \\ R_{\rm sub} &= 15\ \Omega, \\ R_{\rm net} &= 15\ \Omega + 30\ \Omega = 45\ \Omega. \end{aligned} \tag{18}$$

Problem #**4:**

A wire of length L carrying current I in a magnetic field B feels magnetic force

$$\vec{F} = I\vec{L} \times \vec{B} \tag{19}$$

where $\times$ is the cross product of two vectors. The magnitude of this force is

$$F = ILB \sin\theta \tag{20}$$

where θ is the angle between the wire and the magnetic field's direction, the direction of the force is perpendicular to both the the wire and the $\vec{B}$ field, and the choice between two perpendiculars is given by the right hand rule: Place your right hand so that your straight index finger point parallel to the wire in the direction of the current while your bent middle finger points in the direction of the magnetic field, then your stretched-out thumb points in the direction of the force.

(a) For the first wire, $I = 10$ A, $L = 0.50$ m, $B = 0.020$ T, and the direction of the current — horizontally from east to west — is perpendicular to the magnetic field's direction — vertically up. Consequently, the force on this wire has magnitude

$$F_1 \;=\; (10\text{ A}) \times (0.50\text{ m}) \times (0.020\text{ m}) \times \sin 90^\circ \;=\; 0.10\text{ N}. \tag{21}$$

The direction of this force is $\perp$ to both the vertical magnetic field and to the east-west wire, so the force must be along the horizontal north-south axis. To decide between north or south directions, place your right hand palm up — so that your bent middle finger points up with the magnetic field. In this position, your stretched thumb points horizontally to the right of your index finger, so when your index finger points west in the direction of the current, your thumb points north. Thus, the direction of the 0.10 Newton force on the first wire is horizontally due north.

(b) For the second wire, $I = 20$ A, $L = 0.50$ m, $B = 0.020$ T, and the direction of the current — horizontally from south to north — is perpendicular to the magnetic field's direction — vertically up. Consequently, the force on this wire has magnitude

$$F_1 \;=\; (20\text{ A}) \times (0.50\text{ m}) \times (0.020\text{ m}) \times \sin 90^\circ \;=\; 0.20\text{ N}. \tag{22}$$

The direction of this force is $\perp$ to both the vertical magnetic field and to the north-south wire, so the force must be along the horizontal east-west axis. To decide between east or west directions, place your right hand palm up — so that your bent middle finger points up with the magnetic field — while your straight index finger points horizontally due north. In this position, your thumb points to the right of your index finger, *i.e.*, north. Thus, the direction of the 0.20 Newton force on the second wire is horizontally due east.

(c) The third wire is parallel to the magnetic field — both are vertical — and hence feels no magnetic force at all, $F_3 = 0$. Indeed, in eq. (20), $\theta = 0°$, hence $\sin\theta = 0$ and therefore

$$F_3 \;=\; ILB\,\sin\theta = 0 \tag{23}$$

regardless of the magnitudes of the magnetic field or the current.

Problem **#5:**

By Faraday's law, the EMF induced in the coil is the rate of change

$$\mathcal{E} \;=\; \frac{\Delta\Phi}{\Delta t} \tag{24}$$

of the magnetic flux Φ through the coil. The flux is given by the formula

$$\Phi \;=\; B \times NA\,\sin\theta \tag{25}$$

when B is the magnetic field, N is the number of loops in the coil, A is the area of one loop, and θ is the angle between the loops and the magnetic field.

The coil in question has $N = 200$ round loops of radius $r = 1.0$ cm and hence area

$$A \;=\; \pi r^2 \;=\; 3.14\ \mathrm{cm}^2 \;=\; 3.14\cdot 10^{-4}\ \mathrm{m}^2. \tag{26}$$

The angle between the loops and the magnetic field is held fixed at $\theta = 90°$, thus

$$NA\,\sin\theta \;=\; 6.3\cdot 10^{-2}\ m^2, \tag{27}$$

while the magnitude of the magnetic field increases from $B_1 = 0.2$ T to $B_2 = 2.0$ T. Consequently, the magnetic flux through the coil increases from

$$\Phi_1 \;=\; B_1 \times NA\,\sin\theta \;=\; (0.2\ \mathrm{T}) \times (6.3\cdot 10^{-2}\ \mathrm{m}^2) \;=\; 1.26\cdot 10^{-2}\ \mathrm{T\,m}^2 \tag{28}$$

to

$$\Phi_2 \;=\; B_1 \times NA\,\sin\theta \;=\; (2.0\ \mathrm{T}) \times (6.3\cdot 10^{-2}\ \mathrm{m}^2) \;=\; 12.6\cdot 10^{-2}\ \mathrm{T\,m}^2. \tag{29}$$

To calculate the rate at which the magnetic flux changes with time, we divide the net change

$$\Delta\Phi \;=\; \Phi_2 \;-\; \Phi_1 \;=\; 12.6 \;\cdot 10^{-2}\ \mathrm{T\,m^2} \;-\; 1.26 \cdot 10^{-2}\ \mathrm{T\,m^2} \;\approx\; 1.13 \cdot 10^{-2}\ \mathrm{T\,m^2} \tag{30}$$

by the time $\Delta t = 5.0$ s to make this change, which gives us

$$\frac{\Delta\Phi}{\Delta t} \;=\; \frac{1.13 \cdot 10^{-2}\ \mathrm{T\,m^2}}{5.0\ \mathrm{s}} \;\approx\; 2.6 \cdot 10^{-3}\ \mathrm{T\,m^2/s} \;\equiv\; 2.6 \cdot 10^{-3}\ \mathrm{V}. \tag{31}$$

By Faraday's law (24), the EMF induced in the coil during this change is

$$\mathcal{E} \;=\; \frac{\Delta\Phi}{\Delta t} \;=\; 2.6 \cdot 10^{-3}\ \mathrm{V} \tag{32}$$

or 2.6 milliVolts.