Problem **#1:**

(a) In general, a traveling wave has form

$$y(x,t) \;=\; f(x \mp ut) \tag{1}$$

where $f$ is a function of a single variable: $f(x)$ gives the shape of the wave pulse at time $t = 0$. Shifting the argument $x \to x \mp ut$ makes the whole wave travel at speed $u$. The direction of travel depends on the sign: the wave $f(x - ut)$ travels to the right while the wave $f(x + ut)$ travels to the left.

By comparison, a standing wave has general form

$$y(x,t) \;=\; f_1(x) \times f_2(t) \tag{2}$$

where the function $f_1(x)$ depends only on the position but not on the time, while the function $f_2(t)$ depends only on the time but not on the position.

The wave in question can be re-written as

$$y \;=\; 0.01 \times \sin\bigl(2\pi(900 \times t \;-\; 3 \times x)\bigr) \;=\; 0.01 \times \sin\bigl(-6\pi(x \;-\; 300 \times t)\bigr), \tag{3}$$

which clearly has form

$$y(x,t) \;=\; f(x - 300 \times t) \quad \text{for } f(x) \;=\; 0.01 \times \sin(-6\pi \times x). \tag{4}$$

Comparing this formula to eq. (1) we immediately see that *this is a traveling wave.* Also, *it travels to the right* at speed $u = 300$ m/s. (The unit m/s follows from $x$ being in meters and $t$ in seconds.)

(b) The sine function repeats itself with period $2\pi$,

$$\text{for any } \phi,\ \sin(\phi + 2\pi) \;=\; \sin(\phi). \tag{5}$$

Therefore, a sine wave of the form

$$y(t,x) \;=\; A \times \sin\bigl(2\pi f t \;+\; (\text{time-independent})\bigr) \tag{6}$$

repeats in time with period

$$T \;=\; \frac{2\pi}{2\pi f} \;=\; \frac{1}{f} \tag{7}$$

and hence frequency $f$. The wave in question does have the form (6) for $f = 900$, so it is a periodic wave with frequency $f = 900$ Hz. (The unit Hz follows from the time $t$ being in seconds.)

Likewise, a wave having a sinusoidal $x$ dependence of the form

$$\begin{aligned} y(t,x) \;&=\; A \times \sin\bigl(2\pi k x \;+\; (\text{position-independent})\bigr) \\ \text{or } y(t,x) \;&=\; A \times \sin\bigl(-2\pi k x \;+\; (\text{position-independent})\bigr), \end{aligned} \tag{8}$$

repeats in space with wavelength

$$\lambda \;=\; \frac{2\pi}{2\pi k} \;=\; \frac{1}{k}\,. \tag{9}$$

The wave in question indeed has form (8) with $k = 3$ (for $x$ in units of meters), hence its wavelength is

$$\lambda \;=\; \tfrac{1}{3}\ \text{m} \;=\; 0.33\ \text{m}. \tag{10}$$

Finally, the speed of a periodic wave follows from its frequency and wavelength as

$$u \;=\; f \times \lambda \;=\; (900\ \text{Hz}) \times (\tfrac{1}{3}\ \text{m}) \;=\; 300\ \text{m/s}. \tag{11}$$

Naturally, this agrees with the wave speed obtained in part (a). (If it did not agree, we would need to look for a mistake.)

(c) The speed of a transverse wave on a string depends on the string's tension $T$ and linear mass density $m/L$, namely

$$u = \sqrt{\frac{T}{m/L}}. \tag{12}$$

Given the mass density $m/L = 1.00$ g/m $= 1.00 \cdot 10^{-3}$ kg/m and the wave speed $u = 300$ m/s obtained in part (a) or part (b), we can solve eq. (12) for the string tension $T$ as

$$u = \sqrt{\frac{T}{m/L}} \quad \Longrightarrow \quad u^2 = \frac{T}{m/L} \quad \Longrightarrow \quad u^2 \times (m/L) = T, \tag{13}$$

thus

$$T = u^2 \times (m/L) = (300\ \mathrm{m/s})^2 \times (1.00 \cdot 10^{-3}\ \mathrm{kg/m}) = 90\ \mathrm{kg \cdot m/s^2} = 90\ \mathrm{N}. \tag{14}$$

In Anglo–American units, this tension is about 20 pounds.

Problem **#2:**

(a) Any standing sound wave in a pipe must have a node at each end of the pipe. But the type of the node — a pressure node or a displacement node — depends on the pipe end being open or closed. At an open end, the pressure in the pipe equalizes to the outside air pressure, so this must be a pressure node of the sound wave. On the other hand, the air cannot be displaced through a closed end of the pipe, so a closed end must be a displacement node of the sound wave.

For the sound wave in question, we have diagrams of both pressure and displacement amplitudes as functions of $x$. At the left end of the pipe, the pressure has a node (a point of zero amplitude) while the displacement has an antinode (a point of maximal amplitude). This combination indicates that *the left end of the pipe is open.* But at the right end of the pipe, the pressure has an antinode (maximal amplitude) while the displacement has a node (zero amplitude). This combination indicates that *the right end of the pipe is closed.*

(b) The pressure wave diagram shows 4 nodes (one at left end and 3 in the middle) and 4 antinodes (one at right end and 3 in the middle). The antinodes lie at half-points between the nodes, so the whole pipe contains $3\frac{1}{2}$ node-node intervals. In terms of the standing wave's wavelength $\lambda$, the distance between nearby nodes is $\lambda/2$, so the whole pipe must have length

$$L = 3\tfrac{1}{2} \times \frac{\lambda}{2} = \frac{7}{4} \times \lambda. \tag{15}$$

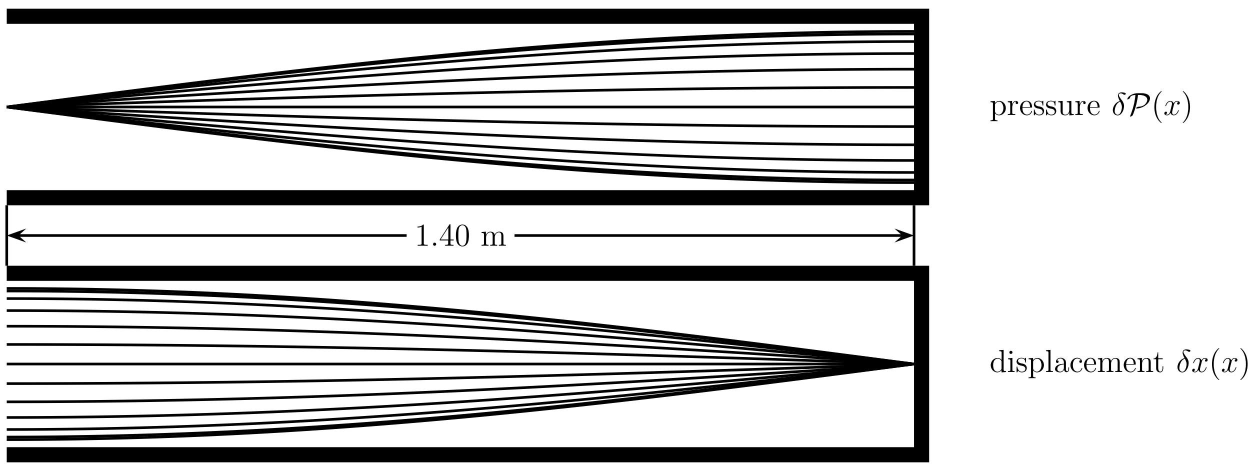
For the wave in question, we do not know the wavelength, but we do know the pipe length $L = 1.40$ m. Consequently, we may solve eq. (15) for the wavelength as

$$\lambda = \frac{4}{7} \times L = \frac{4}{7} \times 1.40 \text{ m} = 0.80 \text{ m}. \tag{16}$$

As to the wave's frequency, it follows from the wavelength and the speed of sound,

$$f = \frac{u}{\lambda} = \frac{336 \text{ m/s}}{0.80 \text{ m}} = 420 \text{ Hz}. \tag{17}$$

(c) All standing sound waves in a pipe must have appropriate nodes at the two ends of the pipe; for the pipe in question, we need a pressure node at the open left end and a displacement node at the closed right end, *cf.* part (a). The fundamental harmonic of the pipe does not have any additional nodes in the middle of the pipe, just one pressure node at the left end and one displacement node at the right end:




Since the displacement node is the antinode of pressure, the distance between it and the node of pressure is one quarter of the wavelength. For the fundamental harmonic, this $\frac{1}{4}\lambda_1$

distance must equal to the whole length $L$ of the pipe, thus

$$\frac{\lambda_1}{4} = L \quad\Longrightarrow\quad \lambda_1 = 4 \times L = 4 \times 1.40 \text{ m} = 5.60 \text{ m}. \tag{18}$$

And the frequency of this fundamental harmonic follows from the wavelength and the speed of sound,

$$f_1 = \frac{u}{\lambda_1} = \frac{336 \text{ m/s}}{5.60 \text{ m}} = 60 \text{ Hz}. \tag{19}$$

PS: The higher harmonics of the pipe have additional pressure and displacement nodes in the middle of the pipe. Consequently, they must fit more $\frac{1}{4}\lambda$ intervals inside the length of the pipe. Specifically, a harmonic $\#n = 2, 3, 4, \ldots$ has

$$L = (2n-1) \times \frac{\lambda_n}{4}, \tag{20}$$

hence wavelength

$$\lambda_n = \frac{4}{2n-1} \times L = \frac{\lambda_1}{2n-1} \tag{21}$$

and frequency

$$f_n = \frac{u}{\lambda_n} = (2n-1) \times \frac{u}{\lambda_1} = (2n-1) \times f_1\,. \tag{22}$$

All such higher harmonics have shorter wavelengths and higher frequencies than the fundamental harmonic. For example, the harmonic #4 analyzed in part (b) has wavelength 7 times shorter and frequency 7 times higher than the fundamental harmonic.

Problem **#3:**

The key to this problem is formula

$$d \times \frac{y}{x} = m \times \lambda \tag{23}$$

at the lower left corner of textbook page 346. Here $d$ is the distance between neighboring lines of the diffraction grating, $x$ is the distance from the grating to the screen, and $y$ is the position of the bright dot $\#m$ on the screen. Note that $m$ must be an integer, $m = 0, \pm1, \pm2, \pm3, \ldots$.

Let me first use this formula to solve the problem, and then I will explain the physical origin of this formula.

(a) Solving eq. (23) for the position of the $m^{\underline{\text{th}}}$ bright dot, we have

$$y_m = m \times \frac{\lambda x}{d}. \tag{24}$$

Note that all such dots are at equal distances

$$\Delta y = \frac{\lambda x}{d} \tag{25}$$

from each other.

Moreover, those distances are proportional to the wavelength $\lambda$ of the light. So we use the same diffraction grating to split beams of light of two different colors — say red and blue — and then project the split beams on the same screen, then the distances between the red dots and the distances between the blue dots will be related to the respective wavelengths as

$$\begin{aligned} \Delta y_{\text{red}} &= \lambda_{\text{red}} \times \frac{x}{d}, \\ \Delta y_{\text{blue}} &= \lambda_{\text{blue}} \times \frac{x}{d}, \end{aligned} \tag{26}$$

where the $x/d$ factor is the same for both colors. Consequently, regardless of the value of that factor, we have

$$\frac{\Delta y_{\text{red}}}{\Delta y_{\text{blue}}} = \frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}} = \frac{633\text{ nm}}{475\text{ nm}} = 1.333. \tag{27}$$

This, given the $\Delta y_{\text{blue}} = 3.00$ cm distances between the blue dots, the distances between the red dots must be

$$\Delta y_{\text{red}} = 1.333 \times \Delta y_{\text{blue}} = 1.333 \times 3.00\text{ cm} = 4.00\text{ cm}. \tag{28}$$

Note that I have solved part (a) without using the distance $x$ to the screen or finding out the line density of the diffraction grating.

(b) The distance $d$ between the lines of the diffraction grating follows from eq. (25):

$$\Delta y \;=\; \frac{\lambda x}{d} \quad\Longrightarrow\quad d \;=\; \frac{\lambda x}{\Delta y}\,. \tag{29}$$

This time we do need to know the distance $x = 1.26$ m from the grating to the screen, and we should use the wavelength $\lambda$ and the dot separation $\Delta y$ for the same color of light. Using those data for the blue light, we obtain

$$d \;=\; \frac{\lambda_{\rm blue}\times x}{\Delta y_{\rm blue}} \;=\; \frac{(475\cdot 10^{-9}\ {\rm m})\times(1.26\ {\rm m})}{3.00\cdot 10^{-2}\ {\rm m}} \;=\; 1.995\cdot 10^{-5}\ {\rm m} \;\approx\; 20.0\ \mu{\rm m}. \tag{30}$$

In other words, the diffraction grating has *line density* of 50 lines per millimeter.

Alternative solution:

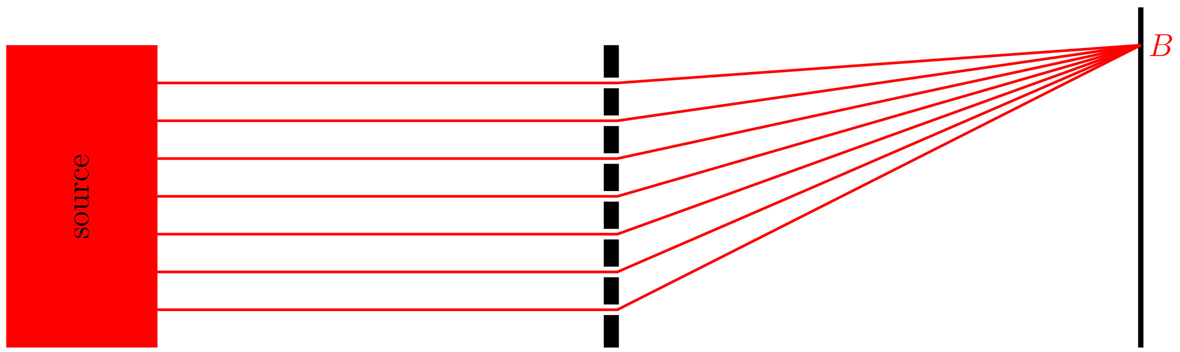
Instead of going through the algebra and deriving the ratio (27), we can simply use eq. (25) twice. First, we use $\lambda_{\rm blue}$, $\Delta y_{\rm blue}$, and the distance to the screen $x$ to solve for $d$ — exactly as we did in part (b) above. Second, given the distance $d$ between the lines, we may use eq. (25) the second time to find $\Delta y$ for the red light,

$$\Delta y_{\rm red} \;=\; \frac{\lambda_{\rm red}\times x}{d} \;=\; \frac{(633\cdot 10^{-9}\ {\rm m})\times(1.26\ {\rm m})}{1.995\cdot 10^{-5}\ {\rm m}} \;=\; 3.998\cdot 10^{-2}\ {\rm m} \;\approx\; 4.00\ {\rm cm}. \tag{31}$$

Explanation of the formula (23):

Note: this explanation is just to help you understand the theory of diffraction gratings. I did no expect you to work this out during the test.

The bright dots on the screen appear at points where all the waves traveling through each slit of the diffraction grating interfere constructively. This requires all the waves getting to the same bright dot $B$ on the screen through different slits



to have travel distances that differ from each other by integer multiples of the wavelength,

$$\text{dist}(\text{source} \to \text{slit}\#n) \ + \ \text{dist}(\text{slit}\#n \to B) \ = \ \text{common} \ + \ \lambda \times \text{integer}\,. \tag{32}$$

For simplicity, let's assume that the light source is directly behind the diffraction but very far away, so the distances from all the slits to the source are approximately the same. This simplifies eq. (32) to

$$\text{dist}(\text{slit}\#n \to B) \ = \ \text{common} \ + \ \lambda \times \text{integer}\,. \tag{33}$$

Also, let's assume that the diffraction grating itself is much smaller than the distance to the screen. Then the differences between the distances from two slits and point $B$ is just the projection of the vector connecting the two slits onto the direction to point $B$,

$$\text{dist}(\text{slit}\#n \to B) \ - \ \text{dist}(\text{slit}\#0 \to B) \ \approx \ -\sin\theta \times (Y_{\text{slit}\#\text{n}} - Y_{\text{slit}\#0}) \tag{34}$$

where $Y$ is the coordinate along the grating and $\theta$ is the angle between the direction towards point $B$ and the perpendicular to the grating. If the grating is parallel to the screen, then

$$\tan\theta \ = \ \frac{y}{x}\,. \tag{35}$$

Since the slits on the diffraction grating are at equal distances from each other, we have

$$Y_{\text{slit}\#\text{n}} \ - \ Y_{\text{slit}\#0} \ = \ n \times d, \tag{36}$$

hence

$$\text{dist}(\text{slit}\#n \to B) \ - \ \text{dist}(\text{slit}\#0 \to B) \ \approx \ -n \times d \times \sin\theta \tag{37}$$

and consequently

$$\text{dist}(\text{slit}\#n \to B) \ - \ \text{dist}(\text{slit}\#n' \to B) \ \approx \ (n' - n) \times d \times \sin\theta. \tag{38}$$

To make sure the waves getting to point $B$ through all the slits satisfy eq. (33) and hence interfere constructively, we need

$$\text{for any two slits}\#n,n' \quad (n'-n)\times d\sin\theta \;=\; \lambda\times\text{some integer.} \tag{39}$$

But since the slit numbers $n, n'$ are themselves integers, all we need is

$$d\sin\theta \;=\; m\lambda \quad \text{for some integer } m. \tag{40}$$

The physical meaning of the integer $m$ here is very simple: it's the number of the narrow beam produced by the diffraction grating, counting from the central beam with $m = 0$ and $\theta = 0$. The other beams have directions

$$\theta_m \;=\; \arcsin\frac{m\times\lambda}{d} \tag{41}$$

and they cross the screen at points

$$y_m \;=\; x\times\tan\theta_m\,. \tag{42}$$

To obtain eq. (23) from these formulae we need one more assumption, namely small angles $\theta_m$ (in units of radians); this assumption is valid when $d \gg \lambda$ and the beam number $m$ is not too large. For small angles, $\cos\theta \approx 1$ and hence $\tan\theta \approx \sin\theta$. Consequently,

$$y_m \;\approx\; x\times\sin\theta_m \;=\; x\times\frac{m\times\lambda}{d} \;=\; m\times\frac{\lambda x}{d}\,,$$

or equivalently

$$d\times\frac{y_m}{x} \;\approx\; m\times\lambda. \tag{23}$$

Problem **#4:**

The microwaves are electromagnetic waves just like the light waves, and they interfere with each other just line the light waves. But the microwaves have wavelengths thousands times longer than light waves, thus to the microwaves, a 1 mm thick sheet of plexiglas looks like a thin film.

So this problem is about the thin-film interference: The microwaves reflected from the two sides of the plexiglas (the air side and the metal side) can either cancel or strengthen each other, just like in the non-textbook problem from the last homework, the light waves reflected from the two sides of the thin layer of oil can either cancel or strengthen each other.

The type of interference between the the two reflected MW waves — destructive or constructive — depends on the difference of their phases

$$\Delta\phi \;=\; 2\pi \times \frac{\Delta s}{\lambda} \tag{43}$$

where $\lambda$ is the MW wavelength and $\Delta s$ is the difference between the distances traveled by each wave. To determine this difference, we note that both reflected waves have to travel from the MW source to the outer surface of the plexiglas, and later from that outer surface to the MW detector where they interfere. In addition, the wave reflected from the plexiglas-metal interface travels *twice* through the plexiglass sheet — once on its way to the metal, and once on the way back — and that's the distance through which the wave reflected from the air-plexiglas interface does not get to travel. Therefore,

$$\Delta s \;=\; 2 \times d \;=\; 2.0 \text{ mm} \tag{44}$$

where $d = 1.0$ mm is the thickness of the plexiglas sheet.

Moreover, since the difference follows from the extra distance one of the waves travels (and the other does not) through the plexiglas, the $\lambda$ in eq. (43) is the wavelength of the microwaves while they travel through the plexiglas rather than the vacuum,

$$\lambda \;=\; \frac{u}{f} \tag{45}$$

rather than $c/f$. Thus, the 40 GHz microwaves have wavelength in plexiglas

$$\lambda_{40} = \frac{160 \cdot 10^6 \text{ m/s}}{40 \cdot 10^9 \text{ Hz}} = 4.0 \cdot 10^{-3} \text{ m} = 4.0 \text{ mm}, \tag{46}$$

while the 80 GHz microwaves have wavelengths in plexiglas

$$\lambda_{80} = \frac{160 \cdot 10^6 \text{ m/s}}{80 \cdot 10^9 \text{ Hz}} = 2.0 \cdot 10^{-3} \text{ m} = 2.0 \text{ mm}. \tag{47}$$

It remains to compare these wavelength to the difference (44) between the distances traveled by the two reflected waves. For the 40 GHz microwaves,

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta s}{\lambda} = \frac{2.0 \text{ mm}}{4.0 \text{ mm}} = \frac{1}{2}, \tag{48}$$

so the two reflected waves have opposite phases and interfere destructively. In other words, they cancel each other, so the net reflection of the 40 GHz microwaves from the plexiglas-covered metal door is rather weak.

On the other hand, for the 80 GHz microwaves,

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta s}{\lambda} = \frac{2.0 \text{ mm}}{2.0 \text{ mm}} = 1, \tag{49}$$

so the two reflected waves have similar phases and interfere constructively. In other words, they strengthen each other instead of canceling, so the net reflection of the 80 GHz from the plexiglas-covered metal door is rather strong.

THE SUMMARY:

Covering the metal door with a sheet of plexiglas reduced the reflection of the 40 GHz microwaves, but it did not help with the 80 GHz microwaves.

Problem **#5:**

All kinds of mirrors — flat, convex, or concave — can produce virtual images seeming to be behind the mirror. But different types of mirrors have different relations between distances $d_o$ from the mirror to the object and $d_i$ from the mirror to the image:

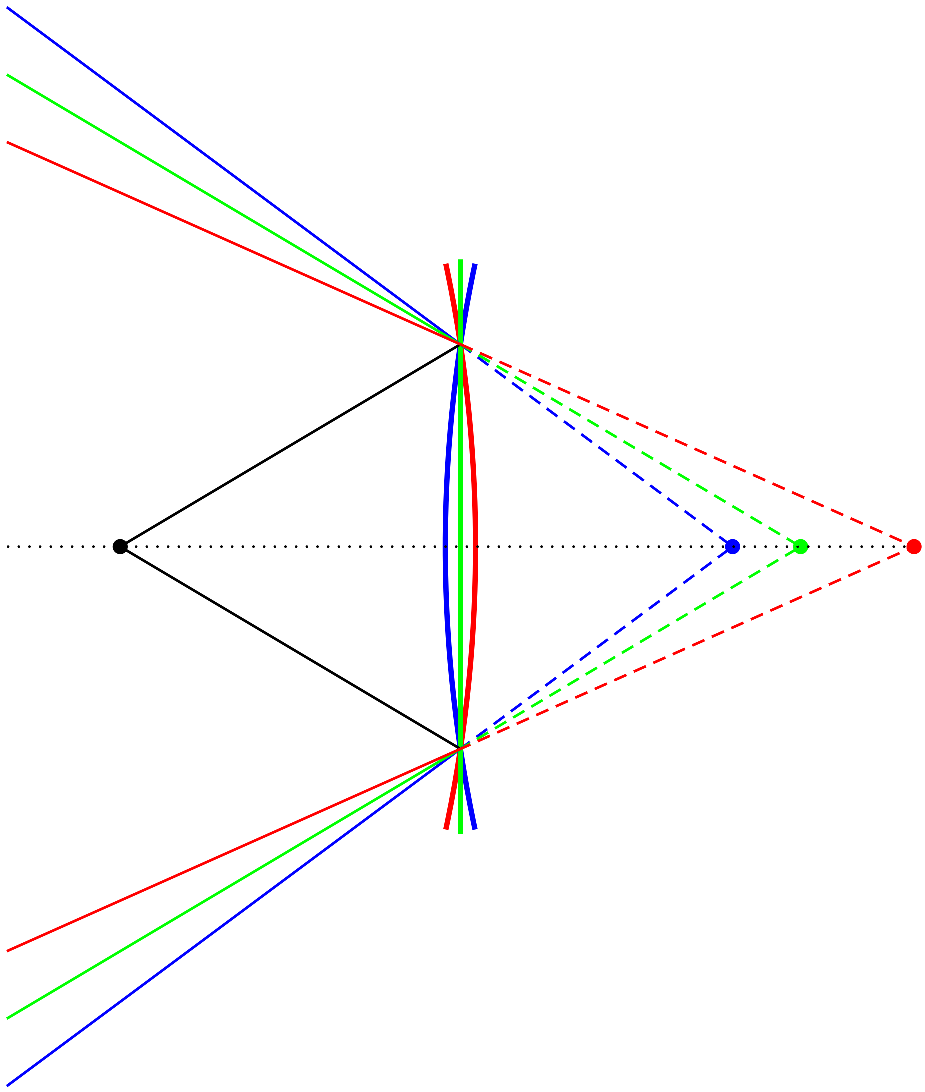
- Flat mirrors do not distort distances: the image seems to be at the same distance behind the mirror as the object is in front of the mirror, $d_i = d_o$.
- Convex mirrors bring images closer to the mirror than the object is, $d_i < d_o$.
- Concave mirrors make the image appear further away then the object, $d_i > d_o$. Now consider the three mirrors in question.

(1) In the first mirror, the image seems to be further away than the object, $d_i = 4'' > d_o = 3''$. According to the above rules, the first mirror must be *concave.*

(2) In the second mirror, the image is at the same distance as the object, $d_i = 4'' = d_o$. This mirror must be *flat.*

(3) In the third mirror, the image is closer in than the object, $d_i = 4'' < d_o = 5''$. This mirror must be *convex.*

To see how the curvature of the mirror affects the image, the following diagram shows rays from the same object reflected by 3 different mirrors: concave, flat, and convex.



On this diagram, the black dot is the object and the black lines are two rays coming out of it. (There is an infinite number of such rays, but I am showing only two.) The thick red, green, and blue lines are the three mirrors, and the thin solid lines are the rays reflected from each mirror. At the points where the rays are reflected, each mirror's surface has a different slope, so the same rays is reflected in different directions by different mirrors. Relative to the vertical flat mirror (green), the concave mirror (red) is curved to the left, so the rays reflected from (red) it go closer to the axis than the rays reflected by the flat mirror (green).

On the other hand, the convex mirror (blue) is curved to the right, so the rays reflected from it (blue) go further away from the axis.

The dotted red, green, and blue lines show the extrapolations of the reflected rays on the other side of the mirrors. The colored dots at the intersection of the extrapolated rays are the images of the object in the three mirrors. Because the concave mirror 'tilts' the reflected rays closer to the axis, their extrapolations (red dashed lines) intersect further away from the mirror, so the image (red dot) if further away from the mirror than the object, $d_i > d_o$. On the other hand, the convex mirror 'tilts' the reflected rays away from the axis, so their extrapolations (blue dashed lines) intersect closer to the mirror. Consequently, the image (blue dot) in the convex mirror is closer to the mirror then the object, $d_i < d_o$.

Finally, let me diagram how three different mirrors can reflect three objects at different distances and get virtual images at the same distance behind the mirror: