Problem **1**:

Consider the car's motion in 3 distinct stages:

(1) Accelerating from zero to $v_f = 30$ m/s. This stage takes time $t_1 = 10$ s, and the car moves through distance

$$L_1 = \tfrac{1}{2}a_1 t_1^2 = \frac{t_1^2}{2} \times \left(a_1 = \frac{v_f}{t_1}\right) = \frac{t_1 \times v_f}{2} = \frac{10\ \text{s} \times 30\ \text{m/s}}{2} = 150\ \text{m}. \tag{S.1}$$

(2) Cruising at constant speed $v = 30$ m/s. During this stage the car moves through $L_2 = 300$ m, which takes time $t_2 = L_2/v = (300\ \text{m})/(30\ \text{m/s}) = 10$ s.

(3) Decelerating from $v_i = 30$ m/s to stop. This stage takes time $t_3 = 5$ s, so the car's acceleration is $a_3 = (0 - v_i)/t_3$ while the car moves through

$$L_3 = v_i t_3 + \tfrac{1}{2}a_3 t_3^2 = v_i t_3 + \frac{t_3^2}{2} \times \frac{-v_i}{t_3} = \frac{v_i \times t_3}{2} = \frac{30\ \text{m/s} \times 5\ \text{s}}{2} = 75\ \text{m}. \tag{S.2}$$

Altogether, the car moves through net distance

$$L_{\text{net}} = L_1 + L_2 + L_3 = 150\ \text{m} + 300\ \text{m} + 75\ \text{m} = 525\ \text{m} \tag{S.3}$$

during net time

$$t_{\text{net}} = t_1 + t_2 + t_3 = 10\ \text{s} + 10\ \text{s} + 5\ \text{s} = 25\ \text{s}. \tag{S.4}$$

Therefore, the *average velocity* of this motion is

$$v_{\text{av}} = \frac{L_{\text{net}}}{v_{\text{net}}} = \frac{525\ \text{m}}{25\ \text{s}} = 21\ \text{m/s}, \tag{S.5}$$

or about 47 MPH.

Problem **2**:

When the car runs past the end of the road and becomes airborne at $t = 0$, its initial velocity v_0 is parallel to the road and hence horizontal. Consequently, at the time t when the car hits the water, its velocity components are

$$\begin{aligned} v_x &= v_0, \\ v_y &= -gt, \end{aligned} \tag{S.6}$$

while the car itself is at

$$x = v_0 t = +30 \text{ m}, \tag{S.7}$$

$$y = -\tfrac{1}{2} g t^2 = -20 \text{ m}. \tag{S.8}$$

(In the coordinate system where the origin is at the end of the road.)

(a) First, we use the vertical equation (S.8) to find the car's time of flight

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{+2 \times 20 \text{ m}}{9.8 \text{ m/s}^2}} = 2.02 \text{ s}. \tag{S.9}$$

Second, we use the horizontal equation (S.7) to find the initial velocity:

$$v_0 = \frac{x}{t} = \frac{30 \text{ m}}{2.02 \text{ s}} = 14.85 \text{ m/s} \approx 15 \text{ m/s}, \tag{S.10}$$

or about 35 MPH.

(b) According to eqs. (S.6), at the time of impact

$$\begin{aligned} v_x &= v_0 = 14.85 \text{ m/s}, \\ v_y &= -gt = -(9.8 \text{ m/s}^2) \times (2.02 \text{ s}) = 19.8 \text{ m/s}. \end{aligned} \tag{S.11}$$

The impact *speed* is the magnitude of this velocity vector, namely

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(14.85 \text{ m/s}^2 + (19.8 \text{ m/s})^2} = 24.75 \text{ m/s} \approx 25 \text{ m/s}, \tag{S.12}$$

or about 56 MPH.

Problem **3**:

Let u_c be the speed of the river current and v_b^w is the speed of the boat relative to the water. The velocity vector $\vec{\mathbf{v}}_b^g$ of the boat relative to the ground (*i.e.*, the river's banks) is

$$\vec{\mathbf{v}}_b^g = \vec{\mathbf{v}}_b^w + \vec{\mathbf{u}}_c\,, \tag{S.13}$$

but the magnitude of this vector — *i.e.*, the ground speed of the boat — depends not only on speeds v_b^w and u_c but also on the angle between the boat's velocity and the current. When the boat travels downstream, the speeds v_b^w and u_c add up, but when the boat travels upstream, they subtract,

$$\begin{aligned} v_b^g[\text{down}] &= v_b^w + u_c\,, \\ v_b^g[\text{up}] &= v_b^w - u_c\,. \end{aligned} \tag{S.14}$$

The boat in question has ground speed (60 km)/(5 h) = 12 km/h when traveling upstream but (60 km)/(3 h) = 20 km/h downstream. In light of equations (S.14), this means

$$v_b^w + u_c = 20\ \text{km/h}, \qquad v_b^w - u_c = 12\ \text{km/h}, \tag{S.15}$$

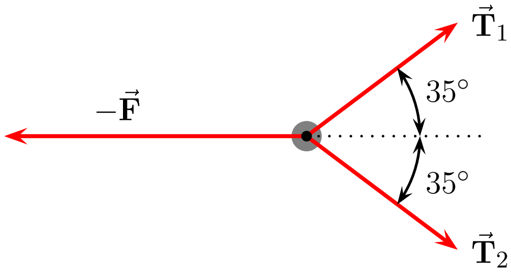
and hence

$$2u_c = \left(\cancel{v_b^w} + u_c\right) - \left(\cancel{v_b^w} - u_c\right) = 20\ \text{km/h} - 12\ \text{km/h} = 8\ \text{km/h}. \tag{S.16}$$

Therefore, the river's current has speed $u_c = 4$ km/h.

Problem **4**:

Consider the forces acting on the middle pulley attached to the leg cast. If that pulley pulls the patient's leg with force $\vec{\mathbf{F}}$, then the leg pulls the pulley back with the force $-\vec{\mathbf{F}}$. The two segments of the wire going around the pulley act on it with forces $\vec{\mathbf{T}}_1$ and $\vec{\mathbf{T}}_2$; these two forces have the same magnitude — the tension T of the wire — but different directions: The $\vec{\mathbf{T}}_1$ pulls 35° above the horizontal and the $\vec{\mathbf{T}}_2$ pulls 35° below the horizontal, as shown on the figure. Neglecting the pulley's own weight, we have



$$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{T}}_1 + \vec{\mathbf{T}}_2 - \vec{\mathbf{F}} \tag{S.17}$$

Since the pulley does not move, its acceleration $\vec{\mathbf{a}}$ is zero, hence by the Second Law $\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = \vec{\mathbf{0}}$ and therefore

$$\vec{\mathbf{F}} = \vec{\mathbf{T}}_1 + \vec{\mathbf{T}}_2\,. \tag{S.18}$$

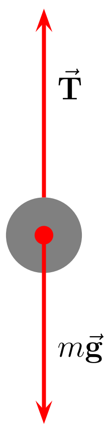
In components,

$$\begin{aligned} F_x &= T_{1x} + T_{2x} = T \times \cos(35^\circ) + T \times \cos(35^\circ) = T \times 2\cos(35^\circ), \\ F_y &= T_{1y} + T_{2y} = T \times \sin(35^\circ) - T \times \sin(35^\circ) = 0, \end{aligned} \tag{S.19}$$

and hence in magnitude $F = T \times 2\cos(35^\circ)$. So if we want to pull the patient's leg with force $F = 80$ N, then the tension of the wire should be

$$T = \frac{F}{2\cos(35^\circ)} = \frac{80\text{ N}}{2 \times 0.819} \approx 49\text{ N}. \tag{S.20}$$

Finally, consider the lead ball at the lower end of the wire: Gravity pulls it down with force $M\vec{\mathbf{g}}$ while the wire tension $\vec{\mathbf{T}}$ pulls it up:



$$M\vec{\mathbf{a}}_{\text{ball}} = \vec{\mathbf{F}}^{\text{net}}_{\text{on ball}} = \vec{\mathbf{T}} + M_{\vec{\mathbf{g}}} \tag{S.21}$$

The ball hands in place without motion, $\vec{\mathbf{a}}_{\text{ball}} = \vec{\mathbf{0}}$, hence $\vec{\mathbf{T}} = -M\vec{\mathbf{g}}$ and the magnitude T of the wire's tension is equal to the lead ball's weight Mg. But the tension is the same throughout the wire, so in order to get this tension as in eq. (S.20), the ball should have weight $Mg = T = 49$ N and hence mass

$$M = \frac{T}{g} = \frac{49\ \text{N}}{9.8\ \text{m/s}^2} \approx 5.0\ \text{kg} = 11\ \text{lb}. \tag{S.22}$$

Problem **5**:

By the Third Law of Newton, the force F pulling John towards Mary has equal magnitude (and opposite direction) as the force pulling Mary towards John. By the Second Law,

$$\begin{aligned} a_J \times m_J &= F^{\text{on John}}, \\ a_M \times m_M &= F^{\text{on Mary}}, \end{aligned} \tag{S.23}$$

and therefore

$$a_J \times m_J = F = a_M \times m_M\,. \tag{S.24}$$

In other words, the product of mass and acceleration is the same for John and for Mary.

Consequently, given both masses and John's acceleration, we may find Mary's acceleration as

$$a_M = \frac{m_J}{m_M} \times a_J = \frac{70\ \text{kg}}{50\ \text{kg}} \times 2.5\ \text{m/s}^2 = 3.5\ \text{m/s}^2. \tag{S.25}$$

PS: Besides the string tension F, there are other forces acting on John and on Mary, namely the forces of gravity and the normal forces from the icy ground. But all such forces are vertical, and do not affect the horizontal motions of Mary and John, and that's why I have ignored them in eqs. (S.23) and (S.24). Also, on slippery ice there are no friction forces, so the only *horizontal* force acting on John or Mary is the string tension F.