Summary of the Standard Model

Setup

The Standard Model of particle physics comprises 12 vector fields, 45 Weyl fermion fields describing 6 quarks and 6 leptons, and a complex doublet H of scalar fields. The vector fields $A^a_\mu(x)$ are gauge fields of the local symmetry

$$G \;=\; SU(3)_C \times SU(2)_W \times U(1)_Y\,; \tag{1}$$

the $SU(3)_C$ mixes 3 *colors* of quarks and antiquarks, the $SU(2)_W$ is the *weak isospin*, and the $U(1)_Y$ couples to the *weak hypercharge* Y. The *electroweak* symmetry $SU(2)_W \times U(1)_Y$ is spontaneously broken by $\langle H\rangle \neq 0$ down to $U(1)_{EM}$; consequently, the $W^\pm$ and Z^0 vector particles become massive ($M_W \approx 80.4$ GeV, $M_Z \approx 91.2$ GeV) while the photon γ remains massless. The photon couples to the *electric charge*

$$q \;=\; T^3 \;+\; Y \tag{2}$$

and gives rise to the electromagnetic interactions; the $W^\pm$ and Z^0 are responsible for the weak interactions. Because photon is massless while the $W^\pm$ and Z^0 are massive, the EM forces have long range while the weak forces are rather short-ranged.

The $SU(3)_C$ vector particles are called *gluons* because they are responsible for strong interactions which "glue" the quarks and antiquarks together into baryons and mesons. At long distances $d \gtrsim 1$ GeV^{-1}, the strong forces become so strong that individual quarks, antiquarks, or gluons cannot be isolated as particles; this phenomenon is called *confinement*. Only the $SU(3)_C$–singlet combinations of quarks, antiquarks, and gluons can be separated from each other.

The quarks come in 3 "colors" $c = 1, 2, 3$ and 6 "flavors" $f = u, d, s, c, b, t$ called 'up', 'down', 'strange', 'charm', 'beauty' (or 'bottom'), and 'truth' (or 'top'); altogether, there are 18 Dirac spinor fields. The left-handed Weyl components of these Dirac spinors form 9 doublets of the $SU(2)_W$ gauge symmetry, 3 flavor doublets $(u, d)_L$, $(c, s)_L$, $(t, b)_L$ for each color. The right-handed Weyl components of the Dirac spinors are all $SU(2)_W$ singlets. This difference between the left-handed and right-handed quarks explains why the weak interactions disrespect the parity symmetry; instead, $W^\pm$ couple to the left currents $J^\mu_L = V^\mu - A^\mu$ and ignore the right currents $J^\mu_R = V^\mu + A^\mu$.

The leptons — e^-, μ^-, τ^-, and 3 neutrino species — have a similar left-right asymmetry. The left-handed leptons form 3 $SU(2)_W$ doublets $(\nu_e, e^e)_L$, $(\nu_\mu, \mu^-)_L$, $(\nu_\tau, \tau^-)_L$, the right-handed charged leptons e^-_R, μ^-_R, τ^-_R are singlets, and the right-handed neutrinos do not even exist.

A Dirac spinor field Ψ and its conjugate $\overline{\Psi}$ are equivalent to two left-handed Weyl spinors χ and $\tilde{\chi}$ and their right-handed conjugates $\chi^\dagger$ and $\tilde{\chi}^\dagger$; χ and $\chi^\dagger$ describe the left-handed fermion and the right-handed antifermion (*e.g.* e^-_L and e^+_R), while $\tilde{\chi}$ and $\tilde{\chi}^\dagger$ describe the left-handed antifermion and the right-handed fermion (*e.g.* e^+_L and e^-_R). The Standard Model has 21 Dirac spinors (3×6 for quarks and 3 for charged leptons) plus 3 Weyl spinors for the neutrinos; in the Weyl language, this amounts to 45 LH Weyl spinors $\chi_\aleph$ and their hermitian conjugates $\chi^\dagger_\aleph$.

Let's organize these spinors by their $SU(3)_C \times SU(2)_W \times U(1)_Y$ quantum numbers.

- Left-handed quarks form 3 $(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$ multiplets Q_n $(n = 1, 2, 3)$; their hermitian conjugates $Q^\dagger_n$ contain the right-handed antiquarks. When the $SU(2)_W \times U(1)_Y$ symmetry is broken to $U(1)_{EM}$, each Q_n splits into a $Q = +\frac{2}{3}$ quark (u, c, or t) and a $q = -\frac{1}{3}$ quark (d, s, or b), both color-triplets. Covariant derivative of a $Q_n(x)$ field with a color index i, an $SU(2)$ index α, and a suppressed Weyl spinor index is

$$\begin{aligned} D_\mu Q^{i\alpha}_n(x) \;&=\; \partial_\mu Q^{i\alpha}_n(x) \;+\; \frac{ig_3}{2}\sum_{C=1}^{8} G^C_\mu(x) \times \lambda^{C\,i}{}_j\, Q^{j,\alpha}_n(x) \\ &\quad+\; \frac{ig_2}{2}\sum_{a=1}^{3} W^a_\mu(x) \times \tau^{a\,\alpha}{}_\beta\, Q^{i\beta}_n(x) \;+\; \frac{ig_1}{6}\, B_\mu(x) \times Q^{i\alpha}_n(x). \end{aligned} \tag{3}$$

- Left-handed antiquarks $\bar{u}, \bar{c}, \bar{t}$ of charge $q = -\frac{2}{3}$ form 3 $(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$ multiplets U_n; the hermitian conjugates $U^\dagger_n$ contain the right-handed quarks u, c, t of charge $q = +\frac{2}{3}$. Covariant derivative of a $U_n(x)$ field with a color index i is

$$D_\mu U_{ni}(x) \;=\; \partial_\mu U_{ni}(x) \;-\; \frac{ig_3}{2}\sum_{C=1}^{8} G^C_\mu(x) \times U_{nj}(x)\, \lambda^{C\,j}{}_i \;-\; \frac{2ig_1}{3}\, B_\mu(x) \times U_{ni}(x). \tag{4}$$

  Note that $SU(2)$ gauge fields $W^a_\mu(x)$ do not enter into this covariant derivative because the U_{ni} fields are $SU(2)$ singlets. Also, the color index i is a lower rather than an upper index because $U_n \in \bar{\mathbf{3}}$ rather than $\mathbf{3}$ for the $SU(3)_C$, and the gluon fields G^C_μ couple differently to the U_{ni} fields than to the $L^{i\alpha}_n$.

- Left-handed antiquarks $\bar{d}, \bar{s}, \bar{b}$ of charge $q = +\frac{1}{3}$ form 3 $(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})$ multiplets D_n; the hermitian conjugates $D^\dagger_n$ contain the right-handed quarks d, s, b of charge $q = -\frac{1}{3}$. Covariant derivative of a $D_n(x)$ field with a color index i is

$$D_\mu D_{ni}(x) \;=\; \partial_\mu D_{ni}(x) \;-\; \frac{ig_3}{2}\sum_{C=1}^{8} G^C_\mu(x)\times D_{nj}(x)\,\lambda^{C\,j}_{\;\;\;i} \;+\; \frac{ig_1}{3}\,B_\mu(x)\times D_{ni}(x). \quad (5)$$

- Left-handed leptons form 3 $(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ multiplets L_n; the hermitian conjugates $L^\dagger_n$ contain right-handed antileptons. When the $SU(2)_W \times U(1)_Y$ symmetry is broken to $U(1)_{EM}$, each L_n splits into a charged lepton e^-, μ^-, or τ^- and a neutrino ν_e, ν_μ, or ν_τ, all color-singlets. Covariant derivative of a $L_n(x)$ field with an $SU(2)$ index α is

$$D_\mu L^\alpha_n(x) \;=\; \partial_\mu L^\alpha_n(x) \;+\; \frac{ig_2}{2}\sum_{a=1}^{3} W^a_\mu(x)\times \tau^{a\,\alpha}_{\;\;\;\beta}\,L^\beta_n(x) \;-\; \frac{ig_1}{2}\,B_\mu(x)\times L^\alpha_n(x). \quad (6)$$

- The left handed anti-leptons e^+_L, μ^+_L, and τ^+_L are singlets of hypercharge $Y = +1$; collectively, they are $E_n \in (\mathbf{1}, \mathbf{1}, +1)$ $(n = 1, 2, 3)$, while the hermitian conjugates $E^\dagger_n$ are the right-handed leptons e^-_R, μ^-_R, and τ^-_R. Covariant derivative of a $E_n(x)$ field is

$$D_\mu E_n(x) \;=\; \partial_\mu E_n(x) \;+\; ig_1\,B_\mu(x)\,E_n(x). \quad (7)$$

Having described all the fields of the Standard Model, we may now write down the Lagrangian:

$$\begin{aligned}
\mathcal{L} \;=\; & -\tfrac{1}{4}\sum_{C=1}^{8} G^C_{\mu\nu}G^{C\mu\nu} \;-\; \tfrac{1}{4}\sum_{a=1}^{3} W^a_{\mu\nu}W^{a\mu\nu} \;-\; \tfrac{1}{4}\,B_{\mu\nu}B^{\mu\nu} \\
& +\; D^\mu H^\dagger\,D_\mu H \;-\; V(H^\dagger H) \\
& +\; \sum_{n,i,\alpha} iQ^\dagger_{ni\alpha}\bar{\sigma}^\mu D_\mu Q^{i\alpha}_n \;+\; \sum_{n,i} iU^{\dagger i}_n\bar{\sigma}^\mu D_\mu U_{ni} \;+\; \sum_{n,i} iD^{\dagger i}_n\bar{\sigma}^\mu D_\mu D_{ni} \\
& +\; \sum_{n,\alpha} iL^\dagger_{n\alpha}\bar{\sigma}^\mu D_\mu L^\alpha_n \;+\; \sum_n iE_n\bar{\sigma}^\mu D_\mu E_n \\
& +\; \mathcal{L}_{\text{Yukawa}} \;+\; \mathcal{L}_{LLHH}\,.
\end{aligned} \quad (8)$$

Here $G^C_{\mu\nu}$, $W^a_{\mu\nu}$, and $B_{\mu\nu}$ are canonically normalized tension fields for the $SU(3)$, $SU(2)$, and $U(1)$ gauge symmetries,

$$\begin{aligned} G^C_{\mu\nu} &= \partial_\mu G^C_\nu - \partial_\nu G^C_\mu - g_3 f^{CDE} G^D_\mu G^E_\nu , \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_2 F \epsilon^{abc} W^b_\mu W^c_\nu , \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu , \end{aligned} \tag{9}$$

the Weyl indices of the fermionic fields Q, U, D, L, and E are implicit, the covariant derivatives D_μ of those fields are as in eqs. (3) through (7), D_μ of the scalar fields are

$$\begin{aligned} D_\mu H^\alpha(x) &= \partial_\mu H^\alpha(x) + \frac{ig_2}{2} \sum_{a=1}^{3} W^a_\mu(x) \times \tau^{a\,\alpha}{}_\beta \, H^\beta(x) + \frac{ig_1}{2} B_\mu(x) \times H^\alpha(x), \\ D_\mu H^\dagger_\alpha(x) &= \partial_\mu H^\dagger_\alpha(x) - \frac{ig_2}{2} \sum_{a=1}^{3} W^a_\mu(x) \times H^\dagger_\beta(x) \tau^{a\,\alpha}{}_\beta - \frac{ig_1}{2} B_\mu(x) \times H^\dagger_\alpha(x), \end{aligned} \tag{10}$$

because $H \in (\mathbf{1}, \mathbf{2}, +\frac{1}{2})$ of the $SU(3) \times SU(2) \times U(1)$, and the scalar potential is

$$V(H^\dagger H) = \frac{\lambda}{2} (H^\dagger H)^2 + m^2 \, H^\dagger H. \tag{11}$$

The m^2 coefficient is negative, so the Higgs fields develop non-zero vacuum expectation values

$$\langle H^\alpha \rangle = \frac{v}{\sqrt{2}} \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{modulo symmetry,} \quad v = \sqrt{\frac{-2m^2}{\lambda}} \,. \tag{12}$$

Experimentally, we know $v \approx 247$ GeV, but we do not know the values of λ or $-m^2$. The VEV (12) breaks the $SU(2)_W \times U(1)_Y$ symmetry down to $U(1)_{EM}$ and gives the $W^\pm$ and Z^0 gauge fields masses $M_W = \frac{1}{2} g_2 \times v$ and $M_Z = \frac{1}{2}\sqrt{g_2^2 + g_1^2} \times v$. In the process, 3 out of 4 real scalar fields comprising H^α are eaten up by the Higgs mechanism, leaving just one real scalar h — the physical Higgs field. Theoretically, its mass is $m_h = \sqrt{-2m^2} = \sqrt{\lambda} \times v$, but we don't know λ, and the experimentalists are still looking for the Higgs particle; all we know at the moment is $m_h > 114$ GeV.

Yukawa Couplings and Fermion Masses

The Standard Model's Lagrangian (8) does not contain any mass terms form the quark and lepton fields. Indeed, the $SU(3)\times SU(2)\times U(1)$ quantum numbers of the 45 Weyl fields $\chi_\aleph = (Q^{i\alpha}_n, U_{ni}, D_{ni}, L^\alpha_n, E_n)$ do not allow for any gauge-invariant mass terms

$$\mathcal{L}_{\rm mass} \;=\; -\tfrac12\sum_{\aleph,\aleph'} M_{\aleph,\aleph'}\,\chi_\aleph^\top\sigma_2\chi_{\aleph'} \;-\; \tfrac12\sum_{\aleph,\aleph'} M^*_{\aleph',\aleph}\,\chi_{\aleph'}^\dagger\sigma_2\chi_\aleph^*\,. \tag{13}$$

Therefore, *before the spontaneous breakdown of the electroweak symmetry, all quarks and leptons were massless.*

Instead of mass terms, the SM Lagrangian contains Yukawa interactions of the fermions and scalar fields. Back in 1935, Hideki Yukawa conjectured that the strong nuclear forces between protons and neutrons are due to exchanges of virtual scalar particles with $O(100\ \mathrm{MeV})$ masses he called *mesons*. In QFT language, the coupling between the scalar meson field $\Phi(x)$ and the Dirac spinor field $\Psi(x)$ of a proton or neutron has form

$$\mathcal{L} \;\supset\; -g\Phi\overline{\Psi}\Psi. \tag{14}$$

When the earliest discovered mesons — the pions — turned up to be pseudo-scalar rather that true scalars, their coupling to nucleons have to be modified as

$$\mathcal{L} \;\supset\; -g\Phi\overline{\Psi}(i\gamma^5)\Psi \tag{15}$$

(isospin indices suppressed). In terms of the Ψ_L and Ψ_R Weyl components of Dirac spinors, both the scalar and the pseudo-scalar Yukawa couplings take form

$$\mathcal{L} \;\supset\; -g\Phi\Psi_R^\dagger\Psi_L \;-\; g^*\Phi\Psi_L^\dagger\Psi_R\,, \tag{16}$$

with a real g for a scalar Φ and imaginary g for pseudoscalar Φ.

In a generic theory without parity or charge-conjugation symmetries, it's often convenient to re-cast all fermionic degrees of freedom in terms of left-handed Weyl fields $\chi_\aleph(x)$ and their conjugates $\chi^*_\aleph(x)$. The Yukawa couplings of such fermionic fields to scalar fields ϕ_s (real or complex) have form

$$\mathcal{L}_{\text{Yukawa}} = -\tfrac{1}{2}\sum_{s,\aleph,\aleph'} Y_{s,\aleph,\aleph'}(\phi_s \text{ or } \phi_s^\dagger)\,\chi_\aleph^\top\sigma_2\chi_{\aleph'} - \tfrac{1}{2}\sum_{s,\aleph,\aleph'} Y^*_{s,\aleph,\aleph'}(\phi_s^\dagger \text{ or } \phi_s)\,\chi_{\aleph'}^\dagger\sigma_2\chi_\aleph^* \tag{17}$$

where the Yukawa couplings $Y_{s,\aleph,\aleph'} = Y_{s,\aleph',\aleph}$ must be invariant under gauge symmetries and other exact symmetries of the theory.

For the Standard Model, ϕ_s is H^α or $H^\dagger_\alpha$, and the gauge-invariant scalar-fermion-fermion combinations are $H^\dagger LE$, $H^\dagger QD$, HQU, and their hermitian conjugates. Thus, the Yukawa interactions of the Standard Model comprise

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & -\sum_{n,n'} Y^E_{n,n'} H^\dagger_\alpha (L^\alpha_n)^\top \sigma_2 E_{n'} \\ & -\sum_{n,n'} Y^D_{n,n'} H^\dagger_\alpha (Q^{i\alpha}_n)^\top \sigma_2 D_{n'i} \\ & -\sum_{n,n'} Y^U_{n,n'} H^\alpha \epsilon_{\alpha\beta} (Q^{i\alpha}_n)^\top \sigma_2 U_{n'i} \\ & + \text{ Hermitian Conjugates.} \end{aligned} \tag{18}$$

Note: implicit summation over color, $SU(2)$, and Weyl spinor indices. The Weyl indices themselves are implicit (not written); the $(L^\beta_n)^\top$, *etc.*, are transposed with respect to Weyl indices only.

Once the scalar fields develop VEVs (12), the Yukawa couplings give rise to the fermion mass terms

$$\begin{aligned} \mathcal{L}_{\text{mass}} = & -\sum_{n,n'=e,\mu,\tau} M^E_{n,n'}\,(L^2_n)^\top\sigma_2 E_{n'} - \sum_{n,n'=e,\mu,\tau} (M^E_{n,n'})^*\,(L^2_n)^\dagger\sigma_2 E^*_{n'} \\ & -\sum_{n,n'=d,s,b} M^D_{n,n'}\,(Q^{i2}_n)^\top\sigma_2 D_{n'i} - \sum_{n,n'=d,s,b} (M^D_{n,n'})^*\,(Q^{i2}_n)^\dagger\sigma_2 D^*_{n'i} \\ & -\sum_{n,n'=u,c,t} M^U_{n,n'}\,(Q^{i1}_n)^\top\sigma_2 U_{n'i} - \sum_{n,n'=u,c,t} (M^U_{n,n'})^*\,(Q^{i1}_n)^\dagger\sigma_2 U^*_{n'i} \end{aligned} \tag{19}$$

where

$$M^E_{n,n'} = \frac{v}{\sqrt{2}}\times Y^E_{n,n'}\,, \qquad M^D_{n,n'} = \frac{v}{\sqrt{2}}\times Y^D_{n,n'}\,, \qquad M^U_{n,n'} = \frac{v}{\sqrt{2}}\times Y^U_{n,n'} \tag{20}$$

are 3×3 *mass matrices* for fermions of similar charges: $M^E_{n,n'}$ is the mass matrix for charged leptons e, μ, τ, $M^D_{n,n'}$ is the mass matrix for quarks d, s, b of charge $q = -\frac{1}{3}$, and $M^U_{n,n'}$ is the mass matrix for quarks u, c, t of charge $q = +\frac{2}{3}$. Indeed, L^2_n are LH charged leptons and E_n are LH charged antileptons; Q^{i2}_n are LH quarks of charge $q = -\frac{1}{3}$ and D_{ni} are the corresponding LH antiquarks; Q^{i1}_n are LH quarks of charge $q = +\frac{2}{3}$ and U_{ni} are the corresponding LH antiquarks.

To go from mass matrices to particle masses, we need to diagonalize the matrices via unitary field redefinitions

$$L^\alpha_n \;\to\; \sum_{n'} \mathcal{U}^L_{n,n'} L^\alpha_{n'}\,, \qquad E_n \;\to\; \sum_{n'} \mathcal{U}^E_{n,n'} E_{n'}\,, \tag{21}$$

and likewise for the quarks Q_n and antiquarks U_n and D_n. In matrix notations, the redefinition (21) turns the lepton mass matrix M_E into

$$M^E \;\to\; \left(\mathcal{U}^L\right)^* M^E \left(\mathcal{U}^E\right)^\dagger. \tag{22}$$

The only invariants of such redefinitions are eigenvalues of the hermitian matrix $M^\dagger_E M_E$, and one can always find some unitary matrices U_L and U_E that would make the M_E matrix diagonal, with real non-negative eigenvalues,⋆

$$M^E \;\to\; \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}. \tag{23}$$

Consequently, we may combine the re-defined fields (21) into Dirac spinor fields

$$\Psi_e \;=\; \begin{pmatrix} L^2_1 \\ \sigma_2 E^*_1 \end{pmatrix}, \qquad \Psi_\mu \;=\; \begin{pmatrix} L^2_2 \\ \sigma_2 E^*_2 \end{pmatrix}, \qquad \Psi_\tau \;=\; \begin{pmatrix} L^2_3 \\ \sigma_2 E^*_3 \end{pmatrix}, \tag{24}$$

⋆ A theorem of matrix algebra says that for any complex $N \times N$ matrix M, there exist two unitary matrices U and V such that UMV is diagonal, and the diagonal elements are real and non-negative. A similar theorem applies to operators in a Hilbert space.

To prove the theorem, note that $M^\dagger M$ is a hermitian matrix with non-negative eigenvalues. Let $H = (M^\dagger M)^{1/2}$. If H is invertible, then $W = MH^{-1}$ is unitary, (Indeed, $W^\dagger W = (H^{-1}M^\dagger)(MH^{-1}) = H^{-1}H^2H^{-1} = 1$.) But even if H is not invertible, there is a unitary matrix W such that $M = WH$ (but I am not going to prove this here).

H is hermitian matrix, so it can be diagonalized as $H = VDV^{-1}$ where V is unitary and D is diagonal (and the eigenvalues are non-negative because H is a positive square root of $M^\dagger M$). For M, this means $M = WVDV^{-1}$ and hence $(WV)^{-1}MV = D$.

so that kinetic and mass terms for the charged leptons become

$$\begin{aligned}\mathcal{L} \supset \sum_{n=e,\mu,\tau} &\Big(i(L^2_n)^\dagger\bar\sigma^\mu D_\mu L^2_n \;+\; iE^\dagger_n\bar\sigma^\mu D_\mu E_n - \;M^E_n\left((L^2_n)^\top\sigma^2 E_n \;+\; E^\dagger_n\sigma_2(L^2_n)^*\right)\Big) \\ &= \overline{\Psi}_e(i\gamma^\mu D_\mu - m_e)\Psi_e \;+\; \overline{\Psi}_\mu(i\gamma^\mu D_\mu - m_\mu)\Psi_\mu \;+\; \overline{\Psi}_\tau(i\gamma^\mu D_\mu - m_\tau)\Psi_\tau\,.\end{aligned} \tag{25}$$

Similar unitary redefinitions of the Q_n, U_n, and D_n Weyl spinor fields make the M_U and M_D quark mass matrices diagonal and real,

$$\begin{aligned}&Q^{i\alpha}_n \;\to\; \sum_{n'}\mathcal{U}^Q_{n,n'}Q^{i\alpha}_{n'}\,, \qquad U_{n,i} \;\to\; \sum_{n'}\mathcal{U}^U_{n,n'}U_{n',i}\,, \\ &\qquad M^U \;\to\; \left(\mathcal{U}^Q\right)^* M^U \left(\mathcal{U}^D\right)^\dagger \;=\; \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t\end{pmatrix},\end{aligned} \tag{26}$$

$$\begin{aligned}&Q^{i\alpha}_n \;\to\; \sum_{n'}\widetilde{\mathcal{U}}^Q_{n,n'}Q^{i\alpha}_{n'}\,, \qquad D_{n,i} \;\to\; \sum_{n'}\mathcal{U}^D_{n,n'}D_{n',i}\,, \\ &\qquad M^D \;\to\; \left(\widetilde{\mathcal{U}}^Q\right)^* M^D \left(\mathcal{U}^D\right)^\dagger \;=\; \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b\end{pmatrix},\end{aligned} \tag{27}$$

which allows us to package all quarks into Dirac spinor fields

$$\begin{aligned}\Psi^i_u &= \begin{pmatrix} Q^{i,1}_1 \\ \sigma_2(U_{1,i})^*\end{pmatrix}, \qquad \Psi^i_c = \begin{pmatrix} Q^{i,1}_2 \\ \sigma_2(U_{2,i})^*\end{pmatrix}, \qquad \Psi^i_t = \begin{pmatrix} Q^{i,1}_3 \\ \sigma_2(U_{3,i})^*\end{pmatrix}, \\ \Psi^i_d &= \begin{pmatrix} Q^{i,2}_1 \\ \sigma_2(D_{1,i})^*\end{pmatrix}, \qquad \Psi^i_s = \begin{pmatrix} Q^{i,2}_2 \\ \sigma_2(D_{2,i})^*\end{pmatrix}, \qquad \Psi^i_b = \begin{pmatrix} Q^{i,2}_3 \\ \sigma_2(D_{3,i})^*\end{pmatrix},\end{aligned} \tag{28}$$

with kinetic and mass terms

$$\mathcal{L}_{\text{quarks}} \;=\; \sum_{f=u,c,t}\overline{\Psi}_{f,i}(i\gamma^\mu D_\mu - m_f)\Psi^i_f \;+\; \sum_{f=d,s,b}\overline{\Psi}_{f,i}(i\gamma^\mu D_\mu - m_f)\Psi^i_f\,. \tag{29}$$

Note that eqs. (26) and (27) transform the left-handed quarks according to *different* 3×3 matrices $\mathcal{U}^Q \neq \widetilde{\mathcal{U}}^Q$. Consequently, in the mass eigenstate basis, the down, strange, and bottom quarks are no longer the $SU(2)$ partners of respectively up, charm, and top quarks. Instead, the $SU(2)$ doublets are

$$(u,d'), \quad (c,s'), \quad (t,b') \tag{30}$$

where d', s', and b' are linear combinations of the d,s,b quarks,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \qquad V = \mathcal{U}^Q \left(\widetilde{\mathcal{U}}^Q\right)^\dagger \neq 1. \tag{31}$$

The unitary matrix V here — called the Cabibbo–Kobayashi–Maskawa matrix (or CKM matrix) — affects the couplings of quarks to the charged $W^\pm_\mu$ vectors mediating weak interactions.

Weak Currents

In eq. (29) couplings of quarks to gauge bosons hide inside the covariant derivatives D_μ. Let's split those derivatives into $\hat{D}_\mu$ that are covariant with respect to the unbroken $SU(3)_C \times U(1)_{\text{EM}}$ symmetries only and the explicit coupling to the massive gauge fields $W^\pm_\mu$ and Z^0_μ of the broken symmetries, thus

$$\begin{aligned} \mathcal{L}_{\text{quarks}} = & \sum_{f=u,c,t,d,s,b} \sum_i \overline{\Psi}_{f,i} \big(i\gamma^\mu \hat{D}_\mu - m_f\big) \Psi^i_f \\ & - \frac{g_2}{\sqrt{2}} \left(W^+_\mu \times T^{-\mu} + W^-_\mu \times T^{+\mu}\right) - \tilde{g} Z^0_\mu \times T^\mu_Z \end{aligned} \tag{32}$$

where

$$\begin{aligned} \hat{D}_\mu \Psi^i_f(x) &= \partial_\mu(x) + ig_3 \sum_{C=1}^{8} \sum_j G^C_\mu(x) \big(\tfrac{1}{2}\lambda^C\big)^i{}_j \Psi^j_f + \tfrac{2}{3}\, ieA_\mu(x) \Psi^i_f \qquad \text{for } f = u,c,t, \\ \hat{D}_\mu \Psi^i_f(x) &= \partial_\mu(x) + ig_3 \sum_{C=1}^{8} \sum_j G^C_\mu(x) \big(\tfrac{1}{2}\lambda^C\big)^i{}_j \Psi^j_f - \tfrac{1}{3}\, ieA_\mu(x) \Psi^i_f \qquad \text{for } f = d,s,b. \end{aligned} \tag{33}$$

Note that the unitary redefinitions (26) and (27) of the Weyl fields commute with color and electric charges, so the do not affect the action of $\hat{D}_\mu$ on the Weyl fermions. Moreover, we may extend their action to the complete Dirac spinors (28) since their left-handed and right-handed components have exactly the same charges, thus eqs. (33).

But the couplings of the weak fields are more complicated. In the Weyl fermion language

$$T^{a,\mu} = \sum_n Q^\dagger_{n,i,\alpha} \bar\sigma^\mu (\tfrac{1}{2}\tau^a)^\alpha_\beta Q^{i,\beta}_n \tag{34}$$

but only in the original basis where $Q^{i,1}_n$ are $SU(2)$ partners of $Q^{i,2}_n$. Indeed, consider the charged weak currents relating $\alpha = 1$ to $\beta = 2$ or vice verse. Using

$$\tfrac{1}{2}(\tau^1 + i\tau^2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \tfrac{1}{2}(\tau^1 - i\tau^2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \tag{35}$$

we may write the charged currents as

$$\begin{aligned} T^{+\mu} &= T^{1,\mu} + iT^{2,\mu} = \sum_n Q^\dagger_{n,i,2} \bar\sigma^\mu Q^{i,1}_n , \\ T^{-\mu} &= T^{1,\mu} - iT^{2,\mu} = \sum_n Q^\dagger_{n,i,1} \bar\sigma^\mu Q^{i,2}_n . \end{aligned} \tag{36}$$

But when we perform different unitary redefinitions $\mathcal{U}^Q$ and $\widetilde{\mathcal{U}}^Q$ for the $Q^{i,1}_n$ and Q^{i1}_n fields as in eqs. (26) and (27), we end up with

$$\begin{aligned} T^{+\mu} &= \sum_{n,n'} V_{n',n} Q^\dagger_{n',i,2} \bar\sigma^\mu Q^{i,1}_n , \\ T^{+\mu} &= \sum_{n,n'} V^*_{n,n'} Q^\dagger_{n,i,1} \bar\sigma^\mu Q^{i,2}_{n'} , \end{aligned} \tag{37}$$

where $V = (\widetilde{\mathcal{U}}^Q)^\dagger \mathcal{U}^Q$ is the Cabibbo–Kobayashi–Maskawa matrix. In terms of Dirac fermions, the currents (37) become

$$\begin{aligned} T^{+\mu} &= \sum_{f=u,c,t} \sum_{f'=d,s,b} V_{f',f} \overline{\Psi}_{f',i} \gamma^\mu \frac{1-\gamma^5}{2} \Psi^i_f , \\ T^{-\mu} &= \sum_{f=u,c,t} \sum_{f'=d,s,b} V^*_{f',f} \overline{\Psi}_{f,i} \gamma^\mu \frac{1-\gamma^5}{2} \Psi^i_{f'} , \end{aligned} \tag{38}$$

where the $(1-\gamma^5)/2$ factor projects on the left-handed Weyl components only. *Thanks to the CKM matrix elements here, weak decays involving charged currents may change the quark flavors not only within families — $t \to b$, $c \to s$, $d \to u$ — but also across families — $t \to s$, $t \to d$, $b \to c$, $b \to u$, $c \to d$, or $s \to u$.*

But the neutral weak current

$$T_Z^\mu \;=\; T^{3,\mu} \;-\; \sin^2\theta\, J^\mu_{\rm el} \tag{39}$$

does not change quark flavors as long as all 3 families have exactly similar $SU(2)\times U(1)$ quantum numbers. Indeed, the $T_Z = T^3 - \sin^2\theta Q_{\rm el}$ charge matrix is diagonal in the original basis $Q_n^{i,\alpha}, U_{n,i}, D_{n,i}$ of Weyl fermions, hence

$$\begin{aligned} T_Z^\mu \;&= \sum_{\aleph} T_Z(\aleph)\times \chi_\aleph^\dagger \bar\sigma^\mu \chi_\aleph \\ &= \left(+\tfrac12 - \tfrac23\sin^2\theta\right)\times\sum_n Q^\dagger_{n,i,1}\bar\sigma^\mu Q_n^{i,1} \;+\; \left(-\tfrac12 + \tfrac13\sin^2\theta\right)\times\sum_n Q^\dagger_{n,i,2}\bar\sigma^\mu Q_n^{i,2} \\ &\quad+\; \left(0 + \tfrac23\sin^2\theta\right)\times\sum_n U_n^{i\dagger}\bar\sigma^\mu U_n^i \;+\; \left(0 - \tfrac13\sin^2\theta\right)\times\sum_n D_n^{i\dagger}\bar\sigma^\mu D_n^i\,, \end{aligned} \tag{40}$$

and since all the T_Z charges are n-independent, the neutral weak current is unaffected by the unitary field redefinitions (26) and (27). In terms of the Dirac fermions, the neutral current (40) becomes

$$T_Z^\mu \;=\; \sum_{f=u,c,t} \overline\Psi_{f,i}\gamma^\mu\left(+\frac{1-\gamma^5}{4} - \frac23\sin^2\theta\right)\Psi_f^0 \;+\; \sum_{f=d,s,b} \overline\Psi_{f,i}\gamma^\mu\left(-\frac{1-\gamma^5}{4} + \frac13\sin^2\theta\right)\Psi_f^i\,. \tag{41}$$

As promised, this current does not mix flavors, so weak transitions due to the neutral currents never change the flavor.

Note that the absence of flavor-changing neutral currents depends on all quarks of the same electric charge and chirality having the same T^3 and hence T_Z. In a non-standard model, we could have had an un-paired quark whose left-handed and right-handed components are both $SU(2)$ singlets while $Q = Y = -\frac13$. Such a quark would have a different T_Z from the down-type quarks of the same charge, so of the mass matrix had somehow mixed an un-paired quarks with down-type quarks, then in the mass eigenbasis the T_Z charge would have off-diagonal elements. Thus, in a non-standard model like that we would have had flavor-changing neutral current.

Historically, back when only 3 quark flavors u, d, s were known, people assumed the s quark was un-paired. Or rather, (u, d') was an $SU(2)_W$ doublet while s' was a singlet for

$$d' \;=\; d\times\cos\theta_c \;+\; s\times\sin\theta_c\,, \qquad s' = \; s\times\cos\theta_c \;-\; d\times\sin\theta_c\,, \qquad \theta_c \;\approx\; 13^\circ. \tag{42}$$

In such a model, there should be $s \leftrightarrow d$ flavor changing neutral currents, which should have lead to weak decays such as $K^0 \to \mu^+\mu^-$. But experimentally, there are no such decays, nor any other signatures of flavor-changing neutral currents. This made Glashow, Illiopoulos, and Maiani conjecture in 1970 that the s quark (or rather the s') should be a member of a doublet just like the d quark, which means that there must be a fourth quark flavor c to form the (c, s') doublet. And in 1974 this fourth flavor (called 'charm') was experimentally discovered at SLAC and BNL.

Later, when the fifth flavor b was discovered in 1977, most physicists expected it to also be a part of the doublet, so everybody was looking for the sixth flavor t. This expectation turned out to be correct, and the t quark was duly discovered in 1995. The delay was due to very large mass of the top quark, $m_t \approx 173$ GeV, much heavier that the other 5 flavors.

What about the leptonic weak currents? In terms of Weyl fermions L_n^α and E_N,

$$\begin{aligned}
T^{+\mu} &= \sum_n L^\dagger_{n,2}\bar\sigma^\mu L^1_n\,,\\
T^{-\mu} &= \sum_n L^\dagger_{n,1}\bar\sigma^\mu L^2_n\,,\\
T^\mu_Z &= \left(\tfrac12 - 0\right)\sum_n L^\dagger_{n,1}\bar\sigma^\mu L^1_n \;+\; \left(-\tfrac12+\sin^2\theta\right)\sum_n L^\dagger_{n,2}\bar\sigma^\mu L^2_n \;+\; \left(0-\sin^2\theta\right)\sum_n E^\dagger_n\bar\sigma^\mu E_n\,.
\end{aligned} \tag{43}$$

In this formula, we are free to use the mass eigenbasis for the charged leptons e, μ, τ as long as we are using the matching basis for the neutrinos. Thus, if $L_1^2 = e_L^-$, $L_2^2 = \mu_L^-$, and $L_3^2 = \tau_L^-$ then $L_1^1 = \nu_e$ (the electron's neutrino), $L_2^1 = \nu_\mu$ (the muon's neutrino), and $L_3^1 = \nu_\tau$ (the tau's neutrino). For massless neutrinos, the $(\nu_e, \nu_\mu, \nu_\tau)$ basis is as good as any other and better then most. When the neutrino become massive, there is an alternative basis (ν_1, ν_2, ν_3) of mass eigenstates, and one has to consider a CKM-like matrix converting between the two basis. This matrix is important for neutrino oscillations, but in most other experiments involving neutrinos, the weak interactions are more important than the very small neutrino masses, so people stick to the $(\nu_e, \nu_\mu, \nu_\tau)$ basis.

For the charged leptons, we may combine the Weyl spinors L^2_n and E_n into Dirac spinors

$$\Psi_e \;=\; \begin{pmatrix} L^2_1 \\ \sigma_2 E^*_1 \end{pmatrix}, \qquad \Psi_\mu \;=\; \begin{pmatrix} L^2_2 \\ \sigma_2 E^*_2 \end{pmatrix}, \qquad \Psi_\tau \;=\; \begin{pmatrix} L^2_3 \\ \sigma_2 E^*_3 \end{pmatrix}. \tag{24}$$

For the neutrinos, there are only left-handed Weyl spinors $L^1_n(x)$ and no left-handed antineutrinos whose conjugates can serve as independent RH neutrinos. Thus, we do not have Dirac spinor fields for the neutrinos, but we can make turn the Weyl spinors into Majorana spinors as

$$\Psi_{\nu_e} \;=\; \begin{pmatrix} L^1_1 \\ \sigma_2 (L^1_1)^* \end{pmatrix}, \qquad \Psi_{\nu_\mu} \;=\; \begin{pmatrix} L^1_2 \\ \sigma_2 (L^1_2)^* \end{pmatrix}, \qquad \Psi_{\nu_\tau} \;=\; \begin{pmatrix} L^1_3 \\ \sigma_2 (L^1_3)^* \end{pmatrix}. \tag{44}$$

In terms of Dirac spinors for charged leptons and Majorana spinors for neutrinos,

$$\begin{aligned} \mathcal{L}_{\text{leptons}} \;=\; & \sum_{\ell=e,\mu,\tau} \overline{\Psi}_\ell \big(i\gamma^\mu\partial_\mu \,+\, e\gamma^\mu A_\mu \,-\, m_\ell\big)\Psi_\ell \;+\; \sum_{\nu=\nu_e,\nu_\mu,\nu_\tau} \tfrac{1}{2}\overline{\Psi}_\nu (i\gamma^\mu\partial_\mu)\Psi_\nu \\ & -\; \frac{g_2}{\sqrt{2}}\big(W^+_\mu \times T^{-\mu} \,+\, W^-_\mu \times T^{+\mu}\big) \;-\; \tilde{g} Z^0_\mu \times T^\mu_Z \end{aligned} \tag{45}$$

where

$$\begin{aligned} T^{+\mu} \;&=\; \overline{\Psi}_e\,\gamma^\mu\,\frac{1-\gamma^5}{2}\Psi_{\nu_e} \;+\; \overline{\Psi}_\mu\,\gamma^\mu\,\frac{1-\gamma^5}{2}\Psi_{\nu_\mu} \;+\; \overline{\Psi}_\tau\,\gamma^\mu\,\frac{1-\gamma^5}{2}\Psi_{\nu_\tau}\,, \\ T^{-\mu} \;&=\; \overline{\Psi}_{\nu_e}\,\gamma^\mu\,\frac{1-\gamma^5}{2}\Psi_e \;+\; \overline{\Psi}_{\nu_\mu}\,\gamma^\mu\,\frac{1-\gamma^5}{2}\Psi_\mu \;+\; \overline{\Psi}_{\nu_\tau}\,\gamma^\mu\,\frac{1-\gamma^5}{2}\Psi_\tau\,, \\ T^\mu_Z \;&=\; \sum_{\ell=e,\mu,\tau} \overline{\Psi}_\ell\,\gamma^\mu\left(-\frac{1-\gamma^5}{4} + \sin^2\theta\right)\Psi_\ell \;+\; \sum_{\nu=\nu_e,\nu_\mu,\nu_\tau} \overline{\Psi}_\nu\,\gamma^\mu\left(+\frac{1-\gamma^5}{4}\right)\Psi_\nu\,. \end{aligned} \tag{46}$$

Effective Fermi Theory

Consider the massive vector fields $W^\pm_\mu$ and Z^0_μ mediating the weak interactions. The Lagrangian terms for these fields include

$$\begin{aligned} \mathcal{L}_{W,Z} \;=\; & M^2_W\, W^+_\mu W^{-\mu} \;+\; \tfrac{1}{2} M^2_Z\, Z_\mu Z^\mu \;-\; \frac{g_2}{\sqrt{2}}\big(W^+_\mu \times T^{-\mu} + W^-_\mu \times T^{+\mu}\big) \;-\; \tilde{g} Z^0_\mu \times T^\mu_Z \\ & +\; \text{kinetic terms} \;+\; \text{non-abelian interactions} \\ & +\; \text{interactions with the physical Higgs field,} \end{aligned} \tag{47}$$

where the currents $T^{\pm\mu}$ and T^μ_Z include both the quark and the leptonic terms. In low-energy experiments ($E \ll M_W$) there are no physical $W^\pm$ or Z^0 particles, but the response of the $W^\pm_\mu$ and Z_μ fields to the fermionic currents leads to interactions between the fermions.

To see how this works, let's neglect the interactions between the vector fields or their couplings to the physical Higgs field and focus on their couplings to the currents. In this approximation, the vector fields equations of motion become

$$(M_W^2 + \partial^2) W^\pm_\mu \;=\; \frac{g_2}{\sqrt{2}}\, T^\pm_\mu \,, \qquad (M_Z^2 + \partial^2) Z_\mu \;=\; \tilde{g} T^Z_\mu \,. \tag{48}$$

Moreover, at low energies and momenta of all particles, me may ignore the ∂^2 terms on the left hand sides of these formulae compared to the M^2 terms. In this limit,

$$W^\pm_\mu \;=\; \frac{g_2}{\sqrt{2} M_W^2}\, T^\pm_\mu \,, \qquad Z_\mu \;=\; \frac{\tilde{g}}{M_Z^2}\, T^Z_\mu \tag{49}$$

which follow from the effective Lagrangian

$$\mathcal{L}^{\rm eff}_{W,Z} \;\approx\; M_W^2\, W^+_\mu W^{-\mu} + \tfrac{1}{2} M_Z^2\, Z_\mu Z^\mu - \frac{g_2}{\sqrt{2}} \left(W^+_\mu \times T^{-\mu} \,+\, W^-_\mu \times T^{+\mu} \right) - \tilde{g} Z^0_\mu \times T^\mu_Z \,. \tag{50}$$

Furthermore, eqs. (49) are algebraic rather than differential, so we may plug their solutions back into the Lagrangian (50), thus

$$\mathcal{L}^{\rm eff} \;=\; -\frac{g_2^2}{2M_W^2} \times T^+_\mu T^{-\mu} \;-\; \frac{\tilde{g}^2}{2M_Z^2} \times T_{Z,\mu} T^\mu_Z \,. \tag{51}$$

At this point, we have an effective Lagrangian for the interactions between fermionic fields that no longer refers to the massive vector fields we have started from! Instead we have an effective theory of low-energy weak interactions. It's called *the Fermi theory* since Enrico Fermi have written the current $\times$ current effective Lagrangian back in 1930's.

To be precise, Fermi had only the charged currents for the proton, neutron, electron, and the neutrino fields, and the currents were vector currents $V^\mu = \overline{\Psi} \gamma^\mu \Psi$ rather than the left-handed currents $V^\mu - A^\mu$. Over the years, people has added more particles species to the currents, figured out the parity violation in 1950's, conjectured that there might also be a neutral current in 1960's, and eventually discovered its existence in 1970's.

Note: the effective Lagrangian (51) is usually written as

$$\mathcal{L}^{\text{eff}} \;=\; -\frac{G_F}{\sqrt{2}}\left(2T_\mu^+ \times 2T^{-\mu} \;+\; \rho\times(2T_Z)_\mu\times(2T_Z)^\mu)\right) \tag{52}$$

where $G_F = 1.16637\cdot 10^{-5}$ GeV^{-2} is the Fermi's constant known from β decays and ρ is the model dependent relative strength of the neutral-current weak interactions. In the Standard Model

$$G_F \;=\; \frac{\sqrt{2}g_2^2}{8M_W^2} \;=\; \frac{\sqrt{2}/2}{v^2} \quad\Longrightarrow\quad v \;=\; 247\text{ GeV}, \tag{53}$$

and

$$\rho \;=\; \frac{\tilde{g}^2}{M_Z^2}\bigg/\frac{g_2^2}{M_W^2} \;=\; \frac{M_W^2}{M_Z^2}\times\frac{\tilde{g}^2}{g_2^2} \;=\; \cos^2\theta\times\frac{1}{\cos^2\theta} \;=\; 1. \tag{54}$$

Experimentally, ρ is very close to 1, which agrees with the Standard Model and disagrees with many alternative models.

Neutrino Masses

Originally, the Standard Model had exactly massless neutrinos. When the neutrino were experimentally found to oscillate between species $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$ — which calls for small but non-zero neutrino masses — the SM was extended by adding extra couplings to the Lagrangian. The additional couplings were similar to Yukawa couplings but involved two scalar fields instead of one. Specifically, the new couplings — denoted $\mathcal{L}_{LLHH}$ in the Lagrangian (8) — connect two lepton fields to two Higgs fields; in the Weyl fermion language

$$\mathcal{L}_{LLHH} \;=\; \tfrac{1}{2}\sum_{n,n'} N_{n,n'}\left(H^\alpha\epsilon_{\alpha\beta}L_n^\beta\right)^\top\sigma_2\left(H^\alpha\epsilon_{\alpha\beta}L_{n'}^\beta\right) \;+\; \text{Hermitian Conjugates.} \tag{55}$$

Note that the combination $H^\alpha\epsilon_{\alpha\beta}L_n^\beta$ is invariant under all gauge symmetries, but it's a LH Weyl spinor of the Lorentz symmetry, so it needs to be squared to make a good Lagrangian term.

When the Higgs doublet develops VEVs (12), the interactions (55) give rise to the mass terms for the neutrinos $L^1_n = \nu_e, \nu_\mu, \nu_\tau$. Indeed,

$$\left(H^\alpha \epsilon_{\alpha\beta} L^\beta_n\right) \;=\; \frac{v}{\sqrt{2}} \times L^1_n \;+\; (h\nu)\text{ interactions,} \tag{56}$$

hence

$$\mathcal{L}_{\text{LLHH}} \;\supset\; \mathcal{L}_{\nu\,\text{mass}} \;=\; \tfrac{1}{2}\sum_{n,n'} M^\nu_{n,n'}\,(L^1_n)^\top \sigma_2 (L^1_{n'}) \;+\; \tfrac{1}{2}\sum_{n,n'} (M^\nu_{n,n'})^*\,(L^1_n)^\dagger \sigma_2 (L^1_{n'})^* \tag{57}$$

where

$$M^\nu_{n,n'} \;=\; \frac{v^2}{2} \times N_{n,n'}\,. \tag{58}$$

Unlike the dimensionless gauge and Yukawa couplings, the $N_{n,n'}$ couplings have dimensionality $(\text{energy})^{-1}$. We shall see later in class that such couplings make trouble for perturbation theory at high energies, so they are not allowed in UV-complete quantum field theories. However, if the Standard Model is only an effective theory that's valid up to some maximal energy E_{max} but at higher energies must be superseded by a more complete theory, then it's OK for the SM to have *small* negative-dimensionality couplings $N_{n,n'} \leq (1/E_{\text{max}})$. The key word here is *small* — it explains why the neutrinos are much lighter than the other fermions,

$$M^\nu \;<\; \frac{v^2}{E_{\text{max}}} \;\ll\; v. \tag{59}$$

Indeed, for $E_{\text{max}} = O(10^{15}\,\text{GeV})$, this gives us a limit $M^\nu \lesssim 0.1\,\text{eV}$, while the other fermions have masses between 0.51 MeV (the electron) and 170 GeV (the top quark).

In general, the neutrino mass matrix (58) is non-diagonal and complex, although $M^\nu_{n,n'} = M^\nu_{n',n}$. The physical neutrino masses^2 are eigenvalues of the $(M^\nu)^\dagger M^\nu$, and the mass eigenstates ν_1, ν_2, ν_3 could be quite different from the charge-current basis ν_e, ν_μ, ν_τ. Indeed, the experimentally measure neutrino mixing angles are rather large — up to 55°, much larger than the CKM mixing angles for the quarks.