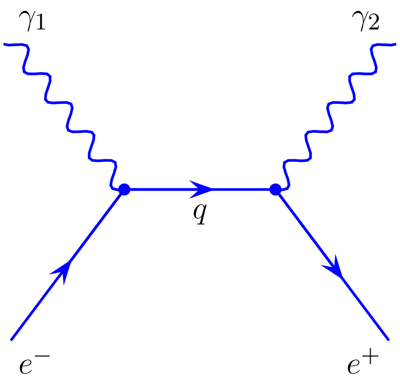
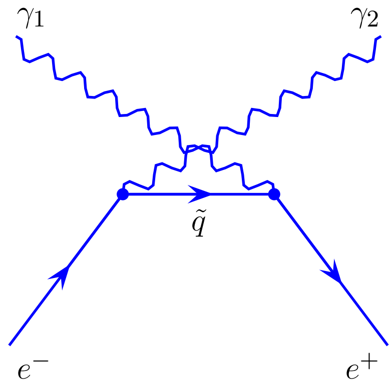
Annihilation *v.* Compton Scattering

Annihilation $e^+e^- \to \gamma\gamma$: 2 tree diagrams related by Bose symmetry:



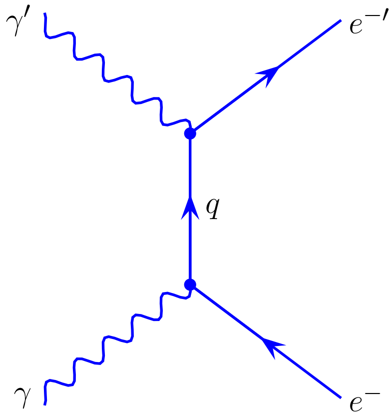
t channel

$+$



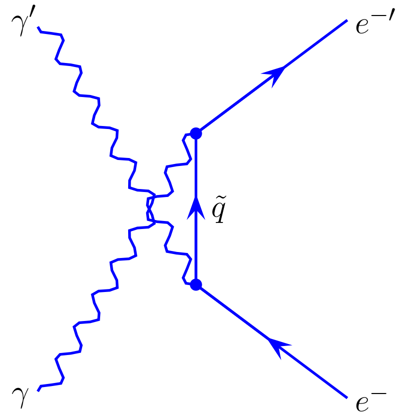
u channel (1)

Compton scattering $\gamma e^- \to \gamma e^-$: 2 similar tree diagrams



s channel

$+$



u channel (2)

The annihilation and the Compton scattering are related by crossing $s \leftrightarrow t$, *i.e.*

$$t^a \leftrightarrow s^c, \qquad s^a \leftrightarrow t^c, \qquad u^a \leftrightarrow u^c. \tag{3}$$

Hence, the amplitudes of these two processes are analytic continuations of each other as functions of momenta,

$$\mathcal{M}^{\text{annihilation}} = f(s,t,u), \qquad \mathcal{M}^{\text{Compton}} = f(t,s,u) \tag{4}$$

for the same analytic function f.

After summing / averaging over fermion spins and photon's polarization, we have

$$\begin{aligned} \sum_{\lambda_1,\lambda_2} \frac{1}{4} \sum_{s_-,s_+} \left| \mathcal{M}^{\text{annihilation}} \right|^2 &= F(s,t,u), \\ \frac{1}{2} \sum_{s,s'} \frac{1}{2} \sum_{\lambda\lambda'} \left| \mathcal{M}^{\text{Compton}} \right|^2 &= -F(t,s,u), \end{aligned} \tag{5}$$

for the same function F. The opposite signs of F in the two formulae follow from crossing an odd number of fermions. These signs are necessary to keep all the spin-summed $|\mathcal{M}|^2$ positive.

To see where such signs come from, consider some generic process $X \to Y$ and let

$$f(s,t,u) = \langle Y | \mathcal{M} | X \rangle, \qquad \bar{f}(s,t,u) = \langle X | \mathcal{M} | Y \rangle. \tag{6}$$

For *real momenta* $\bar{f}$ is the complex conjugate of f, but once we analytically continue to complex momenta, this is no longer true. When we continue to the crossed momenta — which are real but have negative p^0 for the crossed particles — the analytic continuations of the the f and the $\bar{f}$ functions go back to being complex conjugates of each other *up to a sign,*

$$\bar{f} = f^* \times (-1)^{\#\text{FC}} \tag{7}$$

where #FC is the number of fermions we have crossed (*i.e.*, made incoming rather than outgoing or vice verse.)