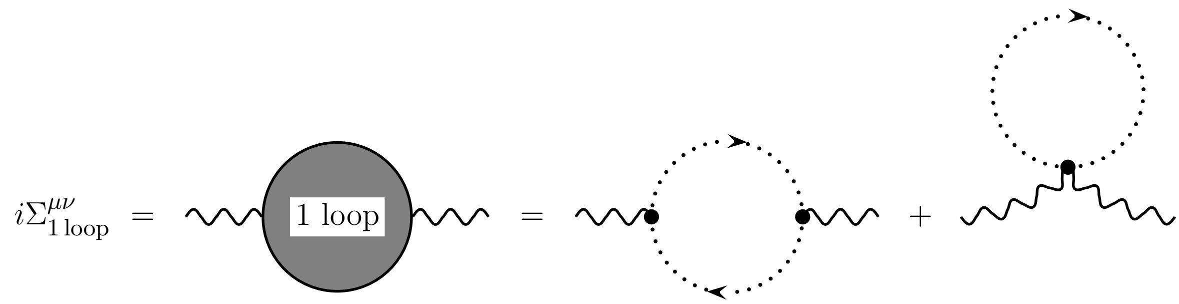
PHY–396 L. Problem set #17. Due February 21, 2013.

1. First, finish the textbook problem **10.2** — calculate to one-loop order the infinite parts of all the counterterms of the pseudoscalar Yukawa theory.

   Hint: the infinite part of the four-scalar amplitude $iV(k_1,\ldots,k_4)$ does not depend on the scalar's momenta, so you may calculate it for any particular $k_1,\ldots,k_4$ you like. The off-shell momenta are OK too, so let $k_1=k_2=k_3=k_4=0$ — this makes for a much easier calculation of the loop diagrams.

Likewise, the infinite part of the one-scalar-two-fermions amplitude $\Gamma^5(p',p)$ does not depend on the momenta, so may calculate it for any p and p' you like, on-shell or off shell; again, the simplest choice is $p=p'=0$.

2. And now consider the electric charge renormalization in the scalar QED — the theory of a EM field A^μ interacting with a charged scalar field Φ. At the one-loop level, there are two Feynman diagrams contributing to the 1PI two-photon amplitude, namely



   (a) Evaluate the two diagrams using dimensional regularization and verify that the net amplitude has form

$$\Sigma^{\mu\nu}_{1\,\text{loop}}(k) \;=\; \left(k^2 g^{\mu\nu} \,-\, k^\mu k^\nu\right) \times \Pi_{1\,\text{loop}}(k^2) \tag{1}$$

(b) Calculate the $\Pi(k^2)$ due to the above diagrams, determine the δ_3 counterterm (at the $O(e^2)$ level), and write down the *net* $\Pi(k^2)$ as a function of k^2.

(c) Finally, consider the effective coupling $\alpha_{\text{eff}}(k^2)$ of the scalar QED at high momenta. Show that at the one-loop level,

$$\frac{1}{\alpha_{\text{eff}}(k^2)} \approx \frac{1}{\alpha(0)} - \frac{1}{12\pi}\left(\log\frac{-k^2}{m^2} - \frac{8}{3}\right). \tag{2}$$