1. First, a simple exercise in using path integrals. Consider a particle on a 1D circle of radius R, or equivalently a 1D particle in a box of length $L = 2\pi R$ with periodic boundary conditions where moving past the $X = L$ points brings you back to $x = 0$. In other words, the particle's position $x(t)$ is defined modulo L.

   (a) Consider all possible particle's paths from a fixed point x_0 (modulo L) at time $t = 0$ to a fixed point x' (modulo L) at time $t = T$. Show that the space of such paths is isomorphic to the space of free particle's paths from a fixed x_0 at $t = 0$ to any of the points $x' + nL$ at $t = T$, for all integer $n = 0, \pm 1, \pm 2, \ldots$. Then use path integrals to show that

$$U_{\text{box}}(x'; x_0) \;=\; \sum_{n=-\infty}^{+\infty} U_{\text{free}}(x' + nL; x_0) \tag{1}$$

   where U_{box} and U_{free} are the evolution kernels (between times $t = 0$ and $t = T$) for the particle in a box and for the free particle.

   The next question uses Poisson's resummation formula: If a function $F(n)$ of integer n can be analytically continued to a function $F(\nu)$ of arbitrary real ν, then

$$\sum_{n=-\infty}^{+\infty} F(n) \;=\; \int\! d\nu\, F(\nu) \times \sum_{n=-\infty}^{+\infty} \delta(\nu - n) \;=\; \sum_{\ell=-\infty}^{+\infty} \int\! d\nu\, F(\nu) \times e^{2\pi i \ell \nu}. \tag{2}$$

   (b) The free particle (living on an infinite 1D line) has evolution kernel

$$U_{\text{free}}(x'; x_0) \;=\; \sqrt{\frac{M}{2\pi i \hbar T}} \times \exp\left(+\frac{iM(x' - x_0)^2}{2\hbar T}\right). \tag{3}$$

   Plug this free kernel into eq. (1) and use Poisson's formula to sum over n.

(c) Verify that the resulting evolution kernel for the particle in a box agrees with the usual QM formula

$$U_{\text{box}}(x';x_0) \;=\; \sum_p L^{-1/2} e^{ipx'/\hbar} \times e^{-iT(p^2/2M)/\hbar} \times L^{-1/2} e^{-ipx_0/\hbar} \tag{4}$$

where p takes box-quantized values

$$p \;=\; \frac{2\pi\hbar}{L} \times \text{integer.} \tag{5}$$

2. Next, a reading assignment: my notes on the properly discretized Euclidean path integral for the harmonic oscillator.

3. Next, the textbook problem **9.2**. Parts (a) and (b) were explained in class and in my notes in the above reading assignment, so you should focus on part (c). Parts (d) and (e) are postponed to the next homework set, after we learn how to integrate over the fermionic and gauge fields.

4. Finally, solve the textbook problem 11.1. In this exercise you should learn why spontaneous breakdown of continuous symmetries does not happen in spacetimes of dimensions $d \leq 2$. Hint: for a massless free scalar field, the coordinate-space formula for the propagator becomes fairly simple. In d Euclidean dimensions,

$$G_0(x-y) \;\equiv\; \int \frac{d^d p_E}{(2\pi)^d} \frac{e^{ip(x-y)}}{p_E^2} \;=\; \frac{\Gamma\left(\frac{d}{2}-1\right)}{4\pi^{d/2}} \times |x-y|^{2-d}, \tag{6}$$

except for $d=2$ where $G_0(x-y) = \text{const} - \frac{1}{2\pi}\log|x-y|$.