1. First, finish the textbook problem **9.2** — solve parts (d) and (e). Also, generalize part (d) to free fermionic fields in $3+1$ dimensions.

2. Next, consider a scalar analogue of QCD, or more generally a theory of Yang–Mills fields A^a_μ and complex scalars Φ_i in some multiplet (r) of the gauge group G.

   (a) Write down the Lagrangian and the Feynman rules of this theory.

   Next, consider the annihilation process $\Phi + \Phi^* \to 2$ gauge bosons. At the tree level, there are four Feynman diagrams contributing to this process.

   (b) Draw the diagrams and write down the tree-level annihilation amplitude.

   As discussed in class, amplitudes involving the non-abelian gauge fields satisfy a weak form of the Ward identity: *On-shell Amplitudes involving* **a** *longitudinally polarized gauge bosons vanish, provided all the other gauge bosons are transversely polarized.* In other words,

$$\begin{aligned} \mathcal{M} \;&\equiv\; e_1^{\mu_1} e_2^{\mu_2} \cdots e_n^{\mu_n}\, \mathcal{M}_{\mu_1\mu_2\cdots\mu_n}(\text{momenta}) \;=\; 0 \\ \text{when}\quad e_1^\mu \propto k_1^\mu \quad &\text{but}\quad e_2^\nu k_{2\nu} \;=\; \cdots \;=\; e_n^\nu k_{n\nu} \;=\; 0. \end{aligned}$$

   (c) Verify this identity for the scalar annihilation amplitude.

3. Finally, a big reading assignment: §12.1 of the *Peskin & Schroeder* textbook about integrating out of the short-distance modes and the Wilsonian renormalization. Also, read §12.4 of the Weinberg's book (vol. 1) about the same subjects.

   These issues are important, and I wish I could spend a lecture or two explaining them. Alas, the class time is too short, hence this assignment.