Wigner and Nambu–Goldstone Modes of Symmetries

In $d \geq 2+1$ dimensions, a continuous symmetry of a quantum field theory — relativistic or not — may be realized in two different modes: the Winger mode (unbroken symmetry) or the Nambu–Goldstone mode (spontaneously broken symmetry).

Wigner Mode (unbroken symmetry)

In the Wigner mode, the ground state $|\text{ground}\rangle$ of the theory — the physical vacuum state of a relativistic theory, or the quasiparticle-vacuum of a condensed matter system — is invariant under the symmetry. Consequently, the charge operators generating the symmetry annihilate the ground state, $\hat{Q}_a |\text{ground}\rangle = 0$. Moreover, the currents $\hat{J}_a^\mu(x)$ also annihilate the ground state, $\hat{J}_a^\mu(x) |\text{ground}\rangle = 0$.

The excited states of a QFT are made by adding particles (or quasiparticles) to the ground state, $|\text{excited}\rangle = \hat{a}^\dagger \cdots \hat{a}^\dagger |\text{ground}\rangle$. For a symmetry realized in a Wigner mode, the particles form multiplets of the symmetry, and all particles in the same multiplet have the same mass (in a relativistic theory); in a non-relativistic theory, all particles in the same multiplet have the same dispersion relation $E(\mathbf{p})$.

Finally, the scattering amplitudes $f(\text{initial particles} \to \text{final particles})$ respect the Wigner-mode symmetries. That is, the amplitudes involving different particles in the same symmetry multiplet are related to each other by symmetry.

Nambu–Goldstone Mode (spontaneously broken symmetry)

In the Nambu–Goldstone mode, the ground state $|\text{ground}\rangle$ is NOT invariant under the symmetry. Instead, there is a continuous family of exactly degenerate ground states, and the symmetry relates them to each other. Consequently, the ground states are NOT annihilated by the symmetry charges or currents, $\hat{Q}_a |\text{ground}\rangle \neq 0$ and $\hat{J}_a^\mu |\text{ground}\rangle \neq 0$.

Furthermore, the particles (or quasiparticles) do NOT form degenerate multiplets of symmetries realized in the Nambu–Goldstone mode, and the scattering amplitudes are NOT symmetric.

Instead, by the **Goldstone Theorem**, the Nambu–Goldstone-mode symmetries have other interesting consequences:

- For every generator $\hat{Q}_a$ of a spontaneously broken symmetry there is a particle species with zero mass. Such particles are called Goldstone particles or Goldstone bosons (since in most cases they are bosons of spin = 0). In non-relativistic theories, the Goldstone particles (or quasiparticles) have energies $E(\mathbf{p})$ that go to zero as $|\mathbf{p}|$ for low momenta, $E(\mathbf{p}) \propto |\mathbf{p}| \to 0$ for $\mathbf{p} \to 0$.
- The currents of broken symmetries create Goldstone particles from the vacuum,

$$\hat{J}^\mu_a(\mathbf{x})\,|\text{ground}\rangle \;\propto\; |1 \text{ Goldstone particle } a \text{ @}\mathbf{x}\rangle\,, \tag{1}$$

or after a Fourier transform from $\mathbf{x}$ to $\mathbf{p}$,

$$\hat{J}^\mu_a(\mathbf{p}) \;=\; \int d^3\mathbf{x}\, L^{-3/2} e^{i\mathbf{p}\mathbf{x}}\, \hat{J}^\mu_a(\mathbf{x}), \tag{2}$$

$$\hat{J}^\mu_a(\mathbf{p})\,|\text{ground}\rangle \;\propto\; |1 \text{ Goldstone particle, species} = a, \text{ momentum} = \mathbf{p}\rangle\,. \tag{3}$$

- The Goldstone particles have the same quantum numbers WRT the *unbroken* symmetries — in particular, they form the same type of a multiplet — as the generators $\hat{Q}_a$ of the broken symmetries.
- Finally, the scattering amplitudes involving low-momentum Goldstone particle vanish as $O(p)$ when the momentum p^μ of the Goldstone particle goes to zero. If multiple Goldstone particles are involved, the amplitude vanishes as $O(p)$ when *any* of the Goldstone particles momenta $\to 0$.

NOTE: A continuous group G of symmetries may be *partially broken* down to a proper subgroup $H \subset G$. That is, the symmetries in H are realized in the Wigner mode while the remaining symmetries in G/H are realized in the Nambu–Goldstone mode. In this case, all particle species — including the Goldstone particles — form degenerate multiplets of H but not of G.

PS: By the **Mermin–Wagner–Coleman Theorem**, in $1+1$ dimensions a continuous symmetry cannot be spontaneously broken. There is no Goldstone mode in $d = 1+1$.