QED Feynman Rules in the Counterterm Perturbation Theory

The simplest version of QED (Quantum ElectroDynamics) has only 2 field types — the EM field A^μ and the electron field Ψ — and the physical Lagrangian

$$\mathcal{L}_{\text{phys}} \;=\; -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \;+\; \overline{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi \;=\; -\tfrac{1}{4}F_{\mu\nu}^2 \;+\; \overline{\Psi}(i\,\partial\!\!\!/ - m)\Psi + \; eA_\mu\overline{\Psi}\gamma^\mu\Psi. \quad (1)$$

Renormalizing the fields according to $A^\mu_{\text{bare}} = \sqrt{Z_3}\times A^\mu$, $\Psi_{\text{bare}} = \sqrt{Z_2}\times\Psi$ and substituting the fields into the *bare* Lagrangian, we obtain

$$\begin{aligned}\mathcal{L}_{\text{bare}} \;&=\; -\frac{Z_3}{4}\times F_{\mu\nu}^2 \;+\; iZ_2\times\overline{\Psi}\,\partial\!\!\!/\Psi \;-\; Z_2 m_{\text{bare}}\times\overline{\Psi}\Psi \\ &\quad+\; \left(Z_1 e \stackrel{\text{def}}{=} Z_2\sqrt{Z_3}e_{\text{bare}}\right)\times A_\mu\overline{\Psi}\gamma^\mu\Psi \\ &=\; \mathcal{L}_{\text{phys}} \;+\; \mathcal{L}_{\text{counterterms}}\end{aligned} \quad (2)$$

where $\mathcal{L}_{\text{phys}}$ is precisely as in eq. (1) while the counterterms comprise

$$\mathcal{L}_{\text{counterterms}} \;=\; -\tfrac{1}{4}\delta_3\times F_{\mu\nu}^2 \;+\; i\delta_2\times\overline{\Psi}\,\partial\!\!\!/\Psi \;-\; \delta_m\times\overline{\Psi}\Psi \;+\; e\delta_1\times A_\mu\overline{\Psi}\gamma^\mu\Psi \quad (3)$$

for

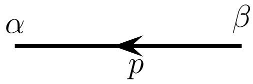
$$\delta_3 \;=\; Z_3 - 1, \quad \delta_2 \;=\; Z_2 - 1, \quad \delta_1 \;=\; Z_1 - 1, \quad \delta_m \;=\; Z_2 m_{\text{bare}} - m_{\text{phys}}\,. \quad (4)$$

In the counterterm perturbation theory, we take the free Lagrangian to be

$$\mathcal{L}_{\text{free}} \;=\; -\tfrac{1}{4}F_{\mu\nu}^2 \;+\; \overline{\Psi}(i\,\partial\!\!\!/ - m)\Psi \quad (5)$$

(where m is the physical mass of the electron) while all the other terms in the bare Lagrangian — the physical coupling $eA_\mu\overline{\Psi}\gamma^\mu\Psi$ and all the counterterms (3) — are treated as perturbations. Consequently, the QED Feynman rules have the following propagators and vertices:

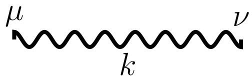
- The electron propagator



$$= \left[\frac{i}{p\!\!\!/ - m + i0}\right]_{\alpha\beta} \;=\; \frac{i(p\!\!\!/ + m)_{\alpha\beta}}{p^2 - m^2 + i0} \quad (6)$$

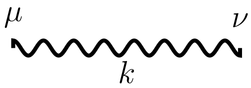
  where α and β are the Dirac indices, usually not written down.

- The photon propagator



$$= \frac{-ig^{\mu\nu}}{k^2 + i0} \tag{7}$$

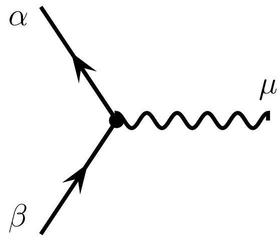
in the Feynman gauge. In a more general Lorentz-invariant gauges, the photon propagator becomes



$$= \frac{-i}{k^2 + i0} \times \left(g^{\mu\nu} + (\xi - 1) \frac{k^\mu k^\nu}{k^2 + i0} \right) \tag{8}$$

for some gauge-dependent parameter ξ.

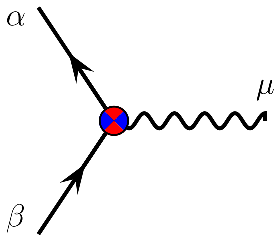
- The physical vertex



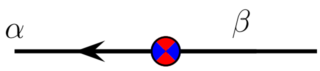
$$= \big(+ie\gamma^\mu \big)_{\alpha\beta} \,. \tag{9}$$

The Dirac indices α and β of the fermionic lines are usually not written down.

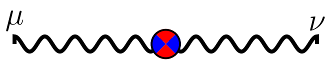
⋆ And then there are three kinds of the counterterm vertices:



$$= +ie\delta_1 \times (\gamma^\mu)_{\alpha\beta} \,, \tag{10}$$



$$= +i\big(\delta_2 \times \not{p} - \delta_m\big)_{\alpha\beta} \,, \tag{11}$$



$$= -i\delta_3 \times \big(g^{\mu\nu}k^2 - k^\mu k^\nu\big). \tag{12}$$