1. Consider the Seiberg duality below the conformal window: The A theory has $N_c^A > \frac{2}{3}N_f$ so it is IR-confining rather than IR-conformal, while the B theory has $N_c^B < \frac{1}{3}N_f$ so it is UV-strong but IR-free. As I explained in class, the B theory emerges as an affective IR theory of the A theory: its "elementary" particles — the $\Phi^{ff'}$ singlets, the quarks and the antiquarks, and even the gluons — are some kinds of composite particles of the confining A theory. The RG flow of the B theory is not asymptotically free but has a Landau pole at some UV scale Λ_B; in terms of the A theory, we expect $\Lambda_B \sim \Lambda_A$.

   Besides the gauge and Yukawa couplings, the B theory may also have non-renormalizable couplings suppressed by the negative powers of the Landau pole, and such couplings may include non-polynomial non-perturbative superpotential terms stemming from some strong interactions at $E \sim \Lambda_B$. Thus, the net superpotential of the B theory should look like

$$W_{\rm net}^B \;=\; \lambda\Phi^{ff'}q_f^c\tilde{q}_{cf'} \;-\; \mu m_{ff'}\Phi^{ff'} \;+\; W_{\rm np}^B(\Phi,\Lambda_B), \tag{1}$$

   where $m_{ff'}$ are the quark masses of the A theory which translate into the O'Raifeartaigh terms of the B theory.

   (a) Suppose all quarks of the A theory are massive but very light, $m \ll \Lambda_A$; in B theory terms, this translates to $\mu m \ll \Lambda_B^2$, so the O'Raifeartaigh terms act as small perturbations. In this regime, the non-perturbative term $W_{\rm np}^B(\Phi,\Lambda_B)$ should have all the flavor symmetries of the $m = 0$ B theory, including the anomalous symmetries which affect the Λ_B.
   Show that these symmetries constrain the form of the non-perturbative term to

$$W_{\rm np}^B(\Phi,\Lambda_B) \;=\; \left(\frac{\det(\lambda\Phi)}{\left(\Lambda^{N_f-3N_c}\right)_B}\right)^{1/N_c^B} \times \text{a numeric constant.} \tag{2}$$

   (b) Show that for ${\rm rank}(m) = N_f$, the B theory with superpotential (1) has $N_f - N_c^B = N_c^A$ supersymmetric vacua. Also show that the $\langle\mu\Phi^{ff'}\rangle$ VEVs in these vacua agree with the meson VEVs $\langle\mu\mathcal{M}^{ff'}\rangle$ of the A theory.

2. The rest of this homeworks is primarily a reading assignment:
   Jeffrey A. Harvey's lecture notes on *Magnetic Monopoles, Duality, and Supersymmetry.* Lectures 1 and 2 should be fairly easy, so please focus on the harder lectures 3, 4, and 5. Skip lecture 6 for now — this material is not germane to this class. (Although it's rather interesting on its own right, so you might want to read lecture 6 when you have more time — or when your own research encounters the multi-monopole moduli spaces.)

   Just for fun, here is a *simple* exercise: Prove the Bogomolny–Prasad–Sommerfeld bound $M \geq v|Q + i\mu|$ for dyons with both electric and magnetic charges.