

Problem #1:

There are two forces acting on the top bead: Its own weight $m_t g$ and the Coulomb force from the bottom bead,

$$F_C = \frac{k q_t q_b}{r^2}. \quad (1)$$

In equilibrium, the two forces cancel each other, which means that they have opposite directions and equal magnitudes

$$|F_C| = \frac{k |q_t| |q_b|}{r^2} = m_t g. \quad (2)$$

Solving this equation for the magnitude of the bottom bead's charge, we find

$$|q_b| = \frac{m_t g \times r^2}{k |q_t|} = \frac{(0.020 \text{ kg}) \times (9.8 \text{ N/kg}) \times (0.30 \text{ m})^2}{(9.0 \cdot 10^9 \text{ N m}^2/\text{C}^2) \times (0.50 \cdot 10^{-6} \text{ C})} \approx 3.9 \cdot 10^{-6} \text{ C}. \quad (3)$$

The sign of the bottom bead's charge follows from the direction of the electrostatic force between the two beads. Equilibrium of the top bead requires the force from the bottom bead to be directed vertically up — otherwise it would not cancel the gravity force — so the two beads should repel each other rather than attract. Since charges of like sign repel while charges of opposite signs attract, it follows that the bottom bead's charge should have the same sign as the top bead's — which happens to be positive.

Altogether, equilibrium of the top bead requires the bottom bead's charge to be $q_b = +3.9 \mu\text{C}$.

Problem #2:

(a) In this series circuit, the same current flows through all the resistors while the voltages add up. Consequently, the resistances of the three resistors add up,

$$R_{\text{net}} = R_1 + R_2 + R_3 = 30 \Omega + 30 \Omega + 30 \Omega = 90 \Omega. \quad (4)$$

(b) In this parallel circuit, the same voltage applies to all the resistors while the currents through them add up. Consequently, the *inverse resistances* of the three resistors add up,

$$\frac{1}{R_{\text{net}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{30 \Omega} + \frac{1}{30 \Omega} + \frac{1}{30 \Omega} = \frac{3}{30 \Omega} = \frac{1}{10 \Omega}, \quad (5)$$

so $R_{\text{net}} = 10 \Omega$.

(c) This circuit has a series sub-circuit of two resistors, and the whole circuit is a parallel connection of that sub-circuit and another resistor. For such ‘nested’ circuit,

$$R_{\text{sub}} = R_1 + R_2 \quad \text{and} \quad \frac{1}{R_{\text{net}}} = \frac{1}{R_{\text{sub}}} + \frac{1}{R_3}. \quad (6)$$

Numerically,

$$\begin{aligned} R_{\text{sub}} &= 30 \Omega + 30 \Omega = 60 \Omega, \\ \frac{1}{R_{\text{net}}} &= \frac{1}{60 \Omega} + \frac{1}{30 \Omega} = \frac{1}{20 \Omega}, \end{aligned} \quad (7)$$

thus $R_{\text{net}} = 20 \Omega$.

(d) Finally, a different kind of nested circuit: It has a parallel sub-circuit of two resistors, and the whole circuit is a series connection of that sub-circuit and another resistor. For this circuit,

$$\frac{1}{R_{\text{sub}}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{and} \quad \frac{1}{R_{\text{net}}} = \frac{1}{R_{\text{sub}}} + \frac{1}{R_3}. \quad (8)$$

Numerically,

$$\begin{aligned} \frac{1}{R_{\text{sub}}} &= \frac{1}{30 \Omega} + \frac{1}{30 \Omega} = \frac{1}{15 \Omega}, \\ R_{\text{sub}} &= 15 \Omega, \\ R_{\text{net}} &= 15 \Omega + 30 \Omega = 45 \Omega. \end{aligned} \quad (9)$$

Problem #3:

(a) The electric arc in the welder obeys Ohm's Law, so at any instance of time the voltage and the current are related as $V(t) = R \times I(t)$. Consequently, for the AC-powered welder, the effective voltage V_{eff} and the effective current I_{eff} — which are respectively the root-mean-square time-averages of the voltage and the current — are also related to each other by the Ohm's Law

$$V_{\text{eff}} = R \times I_{\text{eff}}. \quad (10)$$

Given the effective voltage and the effective current for the welder in question, its electric resistance obtains as

$$R = \frac{V_{\text{eff}}}{I_{\text{eff}}} = \frac{12 \text{ V}}{75 \text{ A}} = 0.16 \text{ } \Omega. \quad (11)$$

(b) By definition of the effective voltage and the effective current, the time-averaged electric power is their product,

$$P = V_{\text{eff}} \times I_{\text{eff}}. \quad (12)$$

For the welder in question, the power is

$$P = 12 \text{ V} \times 75 \text{ A} = 900 \text{ W}. \quad (13)$$

This 900 Watt power means that every second 900 Joules of electric energy are converted to heat. Over the time period of $t = 60$ seconds, the net amount of heat produced by the welder is

$$Q = P \times t = 900 \text{ W} \times 60 \text{ s} = 54\,000 \text{ J}. \quad (14)$$

If you prefer different units for heat, the 54 kiloJoules are equivalent to about 51 BTU (British Thermal Units) or 13 kilocalories. But for this exam, the answer in metric units — 54 kiloJoules or 54000 Joules — is perfectly OK, and you don't have to convert it to any other units.

(c) In a perfect transformer, the AC voltages in two coils have the same ratio as the numbers of turns in the coils,

$$\frac{V_2}{V_1} = \frac{N_1}{N_2}. \quad (15)$$

So if we want a transformer to step $V_1 = 120 \text{ V}$ down to $V_2 = 12 \text{ V}$, the numbers of turns in the two coils must be in ratio

$$\frac{N_1}{N_2} = \frac{V_2}{V_1} = \frac{12 \text{ V}}{120 \text{ V}} = \frac{1}{10}. \quad (16)$$

Thus given $N_1 = 600$ loops in the primary coil, the secondary coil should have

$$N_2 = \frac{1}{10} \times N_1 = \frac{1}{10} \times 600 \text{ turns} = 60 \text{ turns}. \quad (17)$$

(d) While the transformer steps down voltage, it also steps up the current in the same ratio:

$$V_2 = V_1 \times \frac{N_2}{N_1} \quad \text{but} \quad I_2 = I_1 \times \frac{N_1}{N_2}. \quad (18)$$

For the transformer in question $(N_2/N_1) = 1/10$, so

$$V_2 = \frac{V_1}{10} \quad \text{but} \quad I_2 = I_1 \times 10, \quad (19)$$

thus the current through the secondary coil and also through the welder is 10 times the current through the primary coil and hence through the 120 V outlet. Reversing this ratio, we find that the current through the outlet and the primary coil of the transformer is 10 times smaller than the current through the welder and the secondary coil, thus

$$I_2 = I_1 \times 10 \quad \implies \quad I_1 = \frac{I_2}{10} = \frac{75 \text{ A}}{10} = 7.5 \text{ A}. \quad (20)$$

Problem #4:

A wire of length L carrying current I in a magnetic field B feels magnetic force

$$\vec{F} = I\vec{L} \times \vec{B} \quad (21)$$

where \times is the cross product of two vectors. The magnitude of this force is

$$F = ILB \sin \theta \quad (22)$$

where θ is the angle between the wire and the magnetic field's direction, the direction of the force is perpendicular to both the wire and the \vec{B} field, and the choice between two perpendiculars is given by the right hand rule: Place your right hand so that your straight index finger point parallel to the wire in the direction of the current while your bent middle finger points in the direction of the magnetic field, then your stretched-out thumb points in the direction of the force.

(a) For the first wire, $I = 10$ A, $L = 0.50$ m, $B = 0.020$ T, and the direction of the current — horizontally from West to East — is perpendicular to the magnetic field's direction — vertically up. Consequently, the force on this wire has magnitude

$$F_1 = (10 \text{ A}) \times (0.50 \text{ m}) \times (0.020 \text{ T}) \times \sin 90^\circ = 0.10 \text{ N}. \quad (23)$$

The direction of this force is \perp to both the vertical magnetic field and to the East–West wire, so the force must be along the horizontal North–South axis. To decide between North or South directions, place your right hand palm up — so that your bent middle finger points Up with the magnetic field. In this position, your stretched thumb points horizontally to the right of your index finger, so when your index finger points East in the direction of the current, your thumb points South. Thus, the direction of the 0.10 Newton force on the first wire is horizontally due South.

(b) For the second wire, $I = 20$ A, $L = 0.50$ m, $B = 0.020$ T, and the direction of the current — horizontally from South to North — is perpendicular to the magnetic field's direction —

vertically up. Consequently, the force on this wire has magnitude

$$F_1 = (20 \text{ A}) \times (0.50 \text{ m}) \times (0.020 \text{ T}) \times \sin 90^\circ = 0.20 \text{ N}. \quad (24)$$

The direction of this force is \perp to both the vertical magnetic field and to the North–South wire, so the force must be along the horizontal East–West axis. To decide between East or West directions, place your right hand palm up — so that your bent middle finger points up with the magnetic field — while your straight index finger points North. In this position, your thumb points to the right of your index finger, *i.e.*, East. Thus, the direction of the 0.20 Newton force on the second wire is horizontally due East.

(c) The third wire is parallel to the magnetic field — both are vertical — and hence feels no magnetic force at all, $F_3 = 0$. Indeed, in eq. (22), $\theta = 0^\circ$, hence $\sin \theta = 0$ and therefore

$$F_3 = ILB \sin \theta = 0 \quad (25)$$

regardless of the magnitudes of the magnetic field or the current.