Problem **#1:**

(a) In general, a traveling wave has form

$$y(x,t) \;=\; f(x \mp ut) \tag{1}$$

where f is a function of a single variable: $f(x)$ gives the shape of the wave pulse at time $t=0$. Shifting the argument $x \to x \mp ut$ makes the whole wave travel at speed u. The direction of travel depends on the sign: the wave $f(x-ut)$ travels to the right while the wave $f(x+ut)$ travels to the left.

By comparison, a standing wave has general form

$$y(x,t) \;=\; f_1(x) \times f_2(t) \tag{2}$$

where the function $f_1(x)$ depends only on the position but not on the time, while the function $f_2(t)$ depends only on the time but not on the position.

The wave in question can be re-written as

$$y \;=\; 0.01 \times \sin\bigl(2\pi(900 \times t \,-\, 3 \times x)\bigr) \;=\; 0.01 \times \sin\bigl(-6\pi(x \,-\, 300 \times t)\bigr), \tag{3}$$

which clearly has form

$$y(x,t) \;=\; f(x - 300 \times t) \quad \text{for } f(x) \;=\; 0.01 \times \sin(-6\pi \times x). \tag{4}$$

Comparing this formula to eq. (1) we immediately see that *this is a traveling wave.* Also, *it travels to the right* at speed $u = 300$ m/s. (The unit m/s follows from x being in meters and t in seconds.)

(b) The sine function repeats itself with period 2π,

$$\text{for any } \phi,\ \sin(\phi + 2\pi) \ = \ \sin(\phi). \tag{5}$$

Therefore, a sine wave of the form

$$y(t,x) \ = \ A \times \sin\bigl(2\pi f t \ + \ (\text{time-independent})\bigr) \tag{6}$$

repeats in time with period

$$T \ = \ \frac{2\pi}{2\pi f} \ = \ \frac{1}{f} \tag{7}$$

and hence frequency f. The wave in question does have the form (6) for $f = 900$, so it is a periodic wave with frequency $f = 900$ Hz. (The unit Hz for Hertz stand for one cycle per second; it follows from the time t being in seconds.)

Likewise, a wave having a sinusoidal x dependence of the form

$$\begin{aligned} y(t,x) \ &= \ A \times \sin\bigl(2\pi k x \ + \ (\text{position-independent})\bigr) \\ \text{or } y(t,x) \ &= \ A \times \sin\bigl(-2\pi k x \ + \ (\text{position-independent})\bigr), \end{aligned} \tag{8}$$

repeats in space with wavelength

$$\lambda \ = \ \frac{2\pi}{2\pi k} \ = \ \frac{1}{k}\,. \tag{9}$$

The wave in question indeed has form (8) with $k = 3$ (for x in units of meters), hence its wavelength is

$$\lambda \ = \ \tfrac{1}{3}\ \text{m} \ = \ 0.33\ \text{m}. \tag{10}$$

Finally, the amplitude A is the coefficient multiplying the sin function, or rather the absolute value of that coefficient, so that $y(x,t)$ varies in the range from $-A$ to $+A$. In eq. (1) that coefficient is $A = 0.01$ m (since y is in meters) *i.e.* 1 centimeter.

(c) The speed of a periodic wave with known frequency and wavelength obtains as

$$u \;=\; f\times\lambda \;=\; (900\text{ Hz})\times(\tfrac{1}{3}\text{ m}) \;=\; 300\text{ m/s}. \tag{11}$$

Naturally, this agrees with the wave speed obtained in part (a). (If it did not agree, we would need to look for a mistake.)

(d) The speed of a transverse wave on a string depends on the string's tension T and linear mass density m/L, namely

$$u \;=\; \sqrt{\frac{T}{m/L}}\,. \tag{12}$$

Given the mass density $m/L = 1.00\text{ g/m} = 1.00\cdot 10^{-3}\text{ kg/m}$ and the wave speed $u = 300\text{ m/s}$ obtained in part (a) or part (b), we can solve eq. (12) for the string tension T as

$$u \;=\; \sqrt{\frac{T}{m/L}} \quad\Longrightarrow\quad u^2 \;=\; \frac{T}{m/L} \quad\Longrightarrow\quad u^2\times(m/L) \;=\; T, \tag{13}$$

thus

$$T \;=\; u^2\times(m/L) \;=\; (300\text{ m/s})^2\times(1.00\cdot 10^{-3}\text{ kg/m}) \;=\; 90\text{ kg}\cdot\text{m/s}^2 \;=\; 90\text{ N}. \tag{14}$$

In Anglo–American units, this tension is about 20 pounds.

Problem **#2:**

The key to this problem is the formula in the left column of textbook page 346,

$$d\times\frac{y}{x} \;=\; m\times\lambda \tag{15}$$

Here d is the distance between neighboring lines of the diffraction grating, x is the distance from the grating to the screen, and y is the position of the bright dot $\#m$ on the screen. Note that m must be an integer, $m = 0, \pm1, \pm2, \pm3, \ldots$.

Let me first use this formula to solve the problem, and then I will explain the physical origin of this formula. Solving eq. (15) for the position of the $m^{\underline{\text{th}}}$ bright dot, we have

$$y_m \;=\; m \times \frac{\lambda x}{d}\,. \tag{16}$$

Note that all such dots are at equal distances

$$\Delta y \;=\; \frac{\lambda x}{d} \tag{17}$$

from each other.

(a) The problem specifies the wavelength $\lambda = 475$ nm $= 475 \cdot 10^{-9}$ m of the laser light, the distance $x = 1.26$ m from the diffraction grating to the screen, and the distance $\Delta y = 3.00$ cm $= 3.00 \cdot 10^{-2}$ m between the right blue dots on the screen. Plugging in all these data into eq. (17) and solving it for the unknown variable d, we obtain

$$d \;=\; \frac{\lambda \times x}{\Delta y} \;=\; \frac{(475 \cdot 10^{-9}\,\text{m}) \times (1.26\ \text{m})}{3.00 \cdot 10^{-2}\,\text{m}} \;=\; 19.95 \cdot 10^{-6}\,\text{m} \;\approx\; 20.0\ \mu\text{m}. \tag{18}$$

In other words, the diffraction grating has lines spaced every 20 micrometers, thus 50 lines per millimeter.

(b) Given the same diffraction grating as in part (a) with $d = 20.0\ \mu$m, the same distance to the screen $x = 1.26$ m, but a red light with longer wavelength $\lambda' = 633$ nm, the distance between the bright red dots on the screen follows from simply plugging all these data into eq. (17):

$$\Delta y' \;=\; \frac{\lambda' \times x}{d} \;=\; \frac{(633 \cdot 10^{-9}\,\text{m}) \times (1.26\ \text{m})}{29.95 \cdot 10^{-6}\ \text{m}} \;=\; 40.03 \cdot 10^{03}\ \text{m} \;\approx\; 4.00\ \text{cm}. \tag{19}$$

Alternative solution:

Eq. (17) tells us that for the same diffraction grating and the same distance to the screen, the distance Δy between the bright dots on the screen is proportional to the wavelength λ. Indeed,

$$\begin{aligned} \Delta y_{\text{red}} &= \lambda_{\text{red}} \times \frac{x}{d}\,, \\ \Delta y_{\text{blue}} &= \lambda_{\text{blue}} \times \frac{x}{d}\,, \end{aligned} \tag{20}$$

where the x/d factor is the same for both colors. Consequently, regardless of the value of that factor, we have

$$\frac{\Delta y_{\text{red}}}{\Delta y_{\text{blue}}} = \frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}} = \frac{633\text{ nm}}{475\text{ nm}} = 1.333. \tag{21}$$

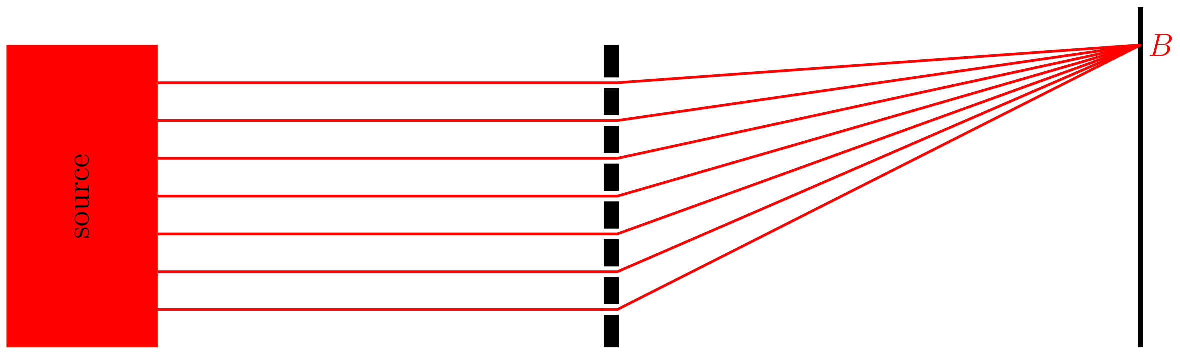
Thus, given the $\Delta y_{\text{blue}} = 3.00$ cm distances between the blue dots, the distances between the red dots must be

$$\Delta y_{\text{red}} = 1.333 \times \Delta y_{\text{blue}} = 1.333 \times 3.00\text{ cm} = 4.00\text{ cm}. \tag{22}$$

Explanation of the formula (15):

Note: this explanation is just to help you understand the theory of diffraction gratings. I did no expect you to work this out during the test.

The bright dots on the screen appear at points where all the waves traveling through each slit of the diffraction grating interfere constructively. This requires all the waves getting to the same bright dot B on the screen through different slits



to have travel distances that differ from each other by integer multiples of the wavelength,

$$\text{dist}(\text{source} \to \text{slit}\#n) \;+\; \text{dist}(\text{slit}\#n \to B) \;=\; \text{common} \;+\; \lambda \times \text{integer}\,. \tag{23}$$

For simplicity, let's assume that the light source is directly behind the diffraction but very far away, so the distances from all the slits to the source are approximately the same. This simplifies eq. (23) to

$$\text{dist}(\text{slit}\#n \to B) \;=\; \text{common} \;+\; \lambda \times \text{integer}\,. \tag{24}$$

Also, let's assume that the diffraction grating itself is much smaller than the distance to the screen. Then the differences between the distances from two slits and point B is just the projection of the vector connecting the two slits onto the direction to point B,

$$\text{dist}(\text{slit}\#n \to B) \;-\; \text{dist}(\text{slit}\#0 \to B) \;\approx\; -\sin\theta \times (Y_{\text{slit}\#\text{n}} - Y_{\text{slit}\#0}) \tag{25}$$

where Y is the coordinate along the grating and θ is the angle between the direction towards point B and the perpendicular to the grating. If the grating is parallel to the screen, then

$$\tan\theta \;=\; \frac{y}{x}\,. \tag{26}$$

Since the slits on the diffraction grating are at equal distances from each other, we have

$$Y_{\text{slit}\#\text{n}} \;-\; Y_{\text{slit}\#0} \;=\; n \times d, \tag{27}$$

hence

$$\text{dist}(\text{slit}\#n \to B) \;-\; \text{dist}(\text{slit}\#0 \to B) \;\approx\; -n \times d \times \sin\theta \tag{28}$$

and consequently

$$\text{dist}(\text{slit}\#n \to B) \;-\; \text{dist}(\text{slit}\#n' \to B) \;\approx\; (n' - n) \times d \times \sin\theta. \tag{29}$$

To make sure the waves getting to point B through all the slits satisfy eq. (24) and hence interfere constructively, we need

$$\text{for any two slits}\#n, n' \quad (n' - n) \times d \sin \theta \;=\; \lambda \times \text{some integer.} \tag{30}$$

But since the slit numbers n, n' are themselves integers, all we need is

$$d \sin \theta \;=\; m\lambda \quad \text{for some integer } m. \tag{31}$$

The physical meaning of the integer m here is very simple: it's the number of the narrow beam produced by the diffraction grating, counting from the central beam with $m = 0$ and $\theta = 0$. The other beams have directions

$$\theta_m \;=\; \arcsin \frac{m \times \lambda}{d} \tag{32}$$

and they cross the screen at points

$$y_m \;=\; x \times \tan \theta_m \,. \tag{33}$$

To obtain eq. (15) from these formulae we need one more assumption, namely small angles θ_m (in units of radians); this assumption is valid when $d \gg \lambda$ and the beam number m is not too large. For small angles, $\cos \theta \approx 1$ and hence $\tan \theta \approx \sin \theta$. Consequently,

$$y_m \;\approx\; x \times \sin \theta_m \;=\; x \times \frac{m \times \lambda}{d} \;=\; m \times \frac{\lambda x}{d} \,,$$

or equivalently

$$d \times \frac{y_m}{x} \;\approx\; m \times \lambda. \tag{15}$$

Problem **#3:**

All kinds of mirrors — flat, convex, or concave — can produce virtual images seeming to be behind the mirror. But different types of mirrors have different relations between distances d_o from the mirror to the object and d_i from the mirror to the image:

- Flat mirrors do not distort distances: the image seems to be at the same distance behind the mirror as the object is in front of the mirror, $d_i = d_o$.

- Convex mirrors bring images closer to the mirror than the object is, $d_i < d_o$.

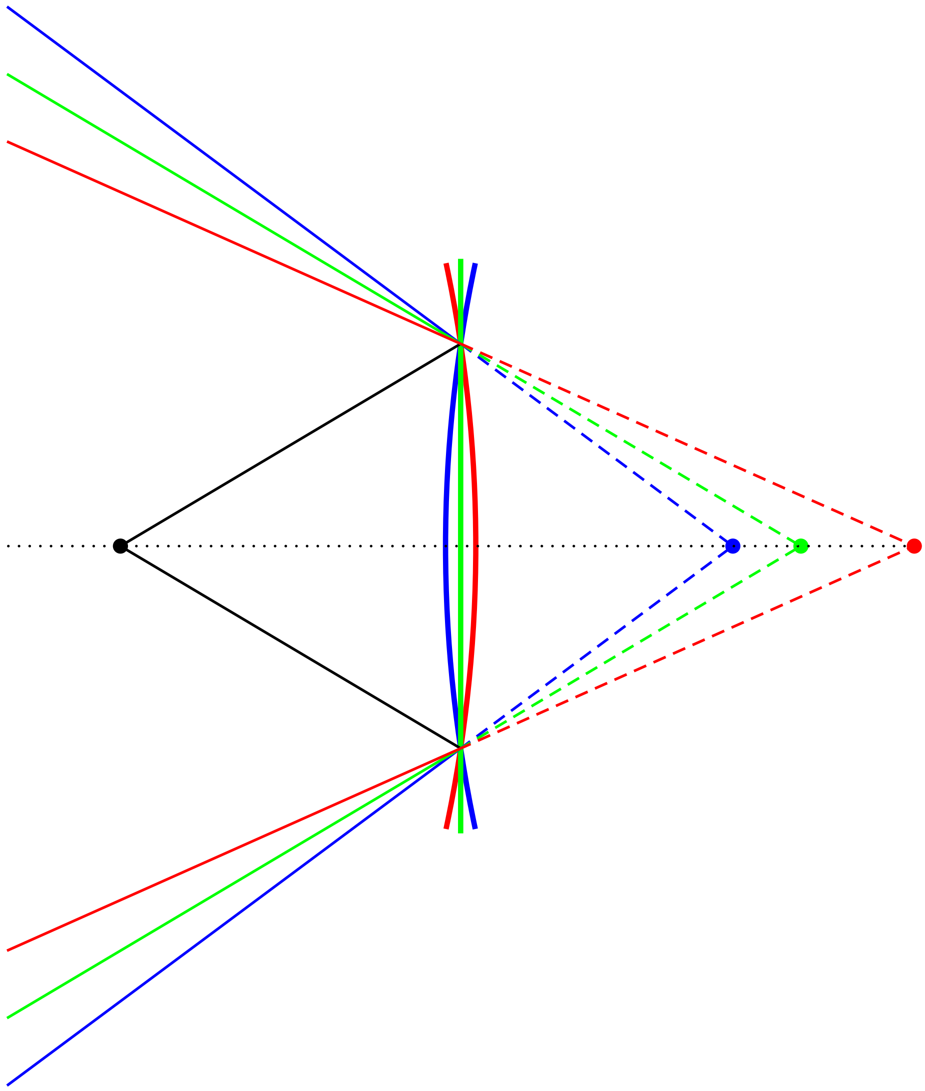
- Concave mirrors make the image appear further away then the object, $d_i > d_o$. Now consider the three mirrors in question.

(1) In the first mirror, the image seems to be further away than the object, $d_i = 4'' > d_o = 3''$. According to the above rules, the first mirror must be *concave.*

(2) In the second mirror, the image is at the same distance as the object, $d_i = 4'' = d_o$. This mirror must be *flat.*

(3) In the third mirror, the image is closer in than the object, $d_i = 4'' < d_o = 5''$. This mirror must be *convex.*

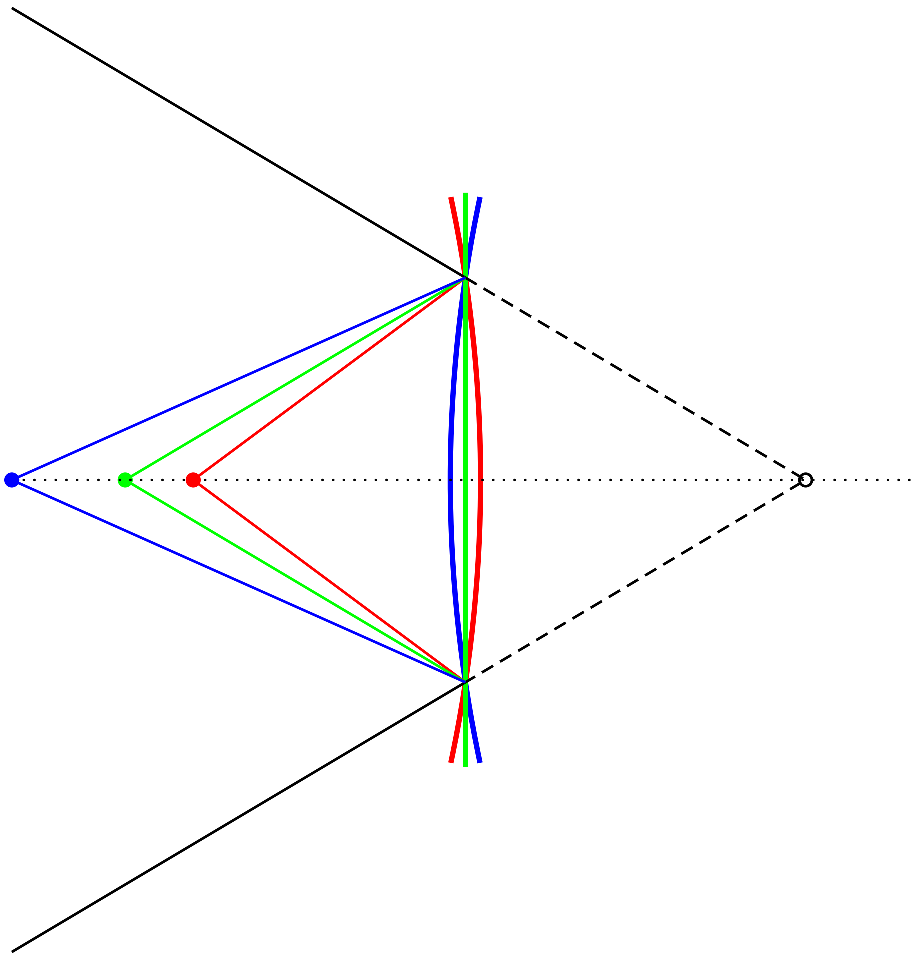
To see how the curvature of the mirror affects the image, the following diagram shows rays from the same object reflected by 3 different mirrors: concave, flat, and convex.



On this diagram, the black dot is the object and the black lines are two rays coming out of it. (There is an infinite number of such rays, but I am showing only two.) The thick red, green, and blue lines are the three mirrors, and the thin solid lines are the rays reflected from each mirror. At the points where the rays are reflected, each mirror's surface has a different slope, so the same rays is reflected in different directions by different mirrors. Relative to the vertical flat mirror (green), the concave mirror (red) is curved to the left, so the rays reflected from (red) it go closer to the axis than the rays reflected by the flat mirror (green). On the other hand, the convex mirror (blue) is curved to the right, so the rays reflected from it (blue) go further away from the axis.

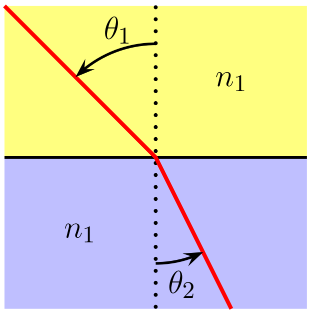
The dotted red, green, and blue lines show the extrapolations of the reflected rays on the other side of the mirrors. The colored dots at the intersection of the extrapolated rays are the images of the object in the three mirrors. Because the concave mirror 'tilts' the reflected rays closer to the axis, their extrapolations (red dashed lines) intersect further away from the mirror, so the image (red dot) if further away from the mirror than the object, $d_i > d_o$. On the other hand, the convex mirror 'tilts' the reflected rays away from the axis, so their extrapolations (blue dashed lines) intersect closer to the mirror. Consequently, the image (blue dot) in the convex mirror is closer to the mirror then the object, $d_i < d_o$.

Finally, let me diagram how three different mirrors can reflect three objects at different distances and get virtual images at the same distance behind the mirror:



Problem **#4:**

When a light ray goes from one transparent medium to another — for example, from air to glass, or from glass to air — its direction changes according to the *Snell's Law of Refraction*

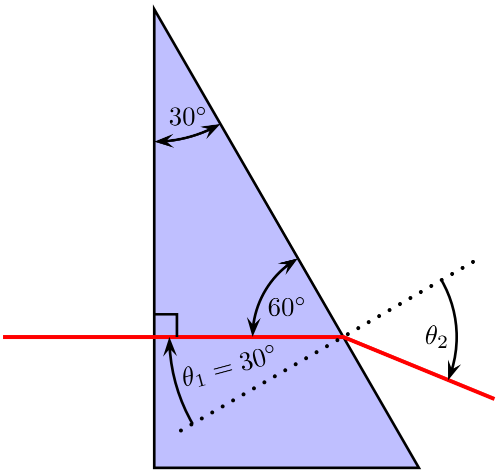


$$n_1 \times \sin\alpha_1 \;=\; n_2 \times \sin\alpha_2\,. \tag{34}$$

Here n_1 and n_2 are the refractive indices of the two media, θ_1 is the angle of incidence, θ_2 is the angle of refraction, and both angles are counted from the perpendicular to the boundary.

The prism in question has vertical left boundary while the light ray before the prism is horizontal, thus $\perp$ to the boundary. This means the angle of incidence is zero, so regardless of the refractive indices the angle of refraction is also zero, and the light ray continues in the horizontal direction inside the prism.

But the right boundary of the prism is tilted at 30° to the vertical, so the horizontal light light ray makes a 60° angle with the boundary, which makes the angle if incidence $\theta_1 = 90° - 60° = 30°$ as shown on the diagram below



This time, the ray does change direction. To find the angle of refraction θ_2, we use the Snell's

Law (34) for refraction from the glass into the air, thus

$$n_1 \;=\; n_{\text{glass}} \;=\; 1.6 \quad\text{and}\quad n_2 \;=\; n_{\text{air}} \;\approx\; 1, \tag{35}$$

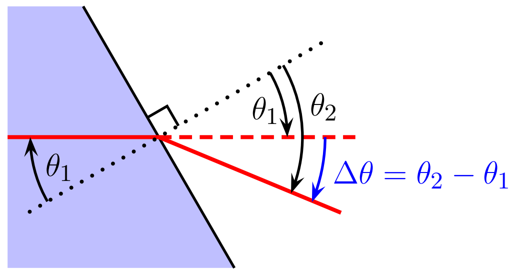
hence

$$\sin\theta_2 \;=\; \frac{n_1}{n_2}\times\sin\theta_1 \;=\; \frac{1.6}{1}\times\sin 30^\circ \;=\; 0.8 \tag{36}$$

and $\theta_2 = \arcsin(0.8) \approx 53^\circ$.

Note that since the glass has a higher refractive index than the air, $n_1 > n_2$, the Snell's Law makes for the angle of refraction to be larger than the angle of incidence, $\theta_2 > \theta_1$. In other words, the light ray bends further away from the $\perp$ to the boundary. which means that for the prism in question, *the light ray bends downward as shown on the left picture* (on page 2 of the problem sheet). This is the answer to part (a) of the problem: *the left picture is right while the right picture is wrong.*

As to part (b), the bending angle $\Delta\theta$ follows from the diagram



Numerically,

$$\Delta\theta \;=\; \theta_2 \;-\; \theta_1 \;=\; 53^\circ \;-\; 30^\circ \;=\; 23^\circ. \tag{37}$$

So the bottom line is: The light ray bends 23° *down.*