

1.7 Accuracy of Physical Data

When physicists collect data in the laboratory, they express their results in a form that includes three essential elements: a number giving the magnitude of the quantity measured, a unit in terms of which the quantity is being measured, and an estimated uncertainty in the measured value, since no physical measurement is ever perfectly accurate. For example, a measurement of the diameter d of a coin with an accurate steel ruler might lead to a value $d = 2.45$ centimeters (cm), where the last figure is an estimate, but the measurement of the same diameter with a precision measuring instrument called a *micrometer caliper* (Fig. 1.14) might yield a value $d = 2.446$ cm. Here the same quantity is measured in the two cases, but the larger number of digits given in the second measurement indicates that it was judged to be more accurate than the first by the person making the measurement.

Significant Figures

The digits given in reporting the results of an experiment, or in stating the data for a problem to be solved, are called *significant figures* (or significant digits). For example, 2.45 cm has three significant figures, whereas 2.446 cm has four. The greater the number of significant figures, the more accurate the data are presumed to be. It is misleading for a physicist or physics student to use more significant figures than are warranted by the apparatus used or the data given in the problem.

Zeros written at the right end of numbers are assumed to be significant figures, for by convention they are included to indicate that the data are certain to this last decimal place. For example, the charge on an electron is 1.60×10^{-19} coulombs (C), where the last zero is a significant figure. Zeros to the left of the first nonzero digit are not significant, since they merely locate the decimal point and say nothing about the accuracy of the number given. Thus 0.00164 is the same as 1.64×10^{-3} in powers-of-10 notation (Appendix 3.B), and in both cases the number has only three significant figures. Because power-of-10 notation makes clear the number of significant figures, it is always used in examples and problems in this book if the number of significant figures would otherwise be unclear.

The following rules are useful in handling significant figures:

- 1 *The final result of an addition or subtraction of two or more quantities should not contain more decimal places than the quantity with the smallest number of decimal places.*



FIGURE 1.14 A micrometer caliper. The instrument is designed so that the movable rod moves 1 millimeter (mm) for each complete turn of the screw. A dial reads fractions of a turn to the nearest 1/100 of a turn. The micrometer can therefore measure distances to 0.01 mm, or 0.001 cm. (Sargent-Welch Scientific Co.)

2 The final result of a multiplication or division should have only as many significant digits as the quantity in the calculation with the smallest number of significant digits.

The basic rationale behind these rules is that if we add an uncertain digit to a certain one, the resulting digit is uncertain and should be omitted from the final sum. Similarly, if a number with two significant figures is multiplied by one with three significant figures, the multiplication of the completely uncertain (actually unknown) third figure of the two-digit number destroys the certainty of the third significant figure in the product. The result can therefore only be given to two significant figures.

The examples and problems in this book have data given in most cases only to two or three significant figures, depending on the nature of the problem. All answers are therefore expected to include two or three significant figures, and no more, for that is all the data can yield. This should be kept in mind when you are tempted to write down the eight digits indicated on your calculator as the answer to a problem or laboratory experiment. Computer programmers have a saying: "Garbage in, garbage out." For our purposes insignificant figures are "garbage." The result of any calculation can be no more certain (contain no more significant figures) than the least certain piece of data used in that calculation.

Example 1.4

How many significant figures are there in the following results for quantities measured in the laboratory?
(a) 2.997924×10^8 m/s **(b)** 3.0120 s **(c)** 0.00124 m **(d)** 100 s

SOLUTION

- (a)** Seven significant figures.
(b) Five significant figures. The final zero is significant, because it indicates that the last decimal place was measured to be zero.
(c) Three significant figures. The zeros are not significant and

are needed only to position the decimal point. For clarity this result would be better written as 1.24×10^{-3} m.

(d) Three significant figures. The final zeros are significant. To make sure that this is what the experimenter intended, it would be clearer if the result were given as 1.00×10^2 s.

Example 1.5

- (a)** Use Table B.2 (inside back cover) to find the mass difference in kilograms between the rest masses of a proton and a neutron.
(b) If seven significant figures are used for the proton and neu-

tron rest masses, how many significant figures does the mass difference have?

SOLUTION

- (a)** This is a case in which many more significant figures than three are required to yield a meaningful result. From Table B.2 we have

$$m_p = 1.672623 \times 10^{-27} \text{ kg} \quad m_n = 1.674929 \times 10^{-27} \text{ kg}$$

The mass difference is then

$$\begin{aligned} \Delta m &= (1.674929 - 1.672623) \times 10^{-27} \text{ kg} \\ &= 0.002306 \times 10^{-27} \text{ kg} \end{aligned}$$

- (b)** The mass difference has only four significant figures and is better written as 2.306×10^{-30} kg. This example illustrates how, after subtraction, the result may have a much smaller number of significant figures than did the original numbers because of cancellations.

Example 1.6

(a) Write 0.00000027 in powers-of-10 notation. **(b)** Calculate $(2.56 \times 10^3) + (7.92 \times 10^4)$. **(c)** Calculate $(2.56 \times 10^3) \times (7.92 \times 10^4)$. **(d)** Calculate $\frac{2.56 \times 10^3}{7.92 \times 10^4}$.

SOLUTION

(a) 2.7×10^{-7}

(b) $(2.56 \times 10^3) + (7.92 \times 10^4) = (2.56 + 79.2) \times 10^3$
 $= 81.8 \times 10^3$

Notice that on adding 2.56 and 79.2, the second decimal place no longer is significant, since 79.2 has only one significant figure to the right of the decimal point. Hence we round off 2.56 to 2.6 and add it to 79.2 to obtain 81.8.

(c) $(2.56 \times 10^3) \times (7.92 \times 10^4) = 20.3 \times 10^7$

Here we multiply 2.56 by 7.92, rounding off the product to three significant figures, and add the powers of 10 to obtain $10^{3+4} = 10^7$.

(d) $\frac{2.56 \times 10^3}{7.92 \times 10^4} = 0.323 \times 10^{-1}$

Here we divide 2.56 by 7.92, round off the quotient to three significant figures, and subtract the powers of 10 to obtain $10^{3-4} = 10^{-1}$. The result can also be written as 3.23×10^{-2} .