5-4 Frictional Forces

We have seen several problems where a body rests or slides on a surface that exerts forces on the body, and we have used the terms *normal force* and *friction force* to describe these forces. Whenever two bodies interact by direct contact (touching) of their surfaces, we call the interaction forces *contact forces.* Normal and friction forces are both contact forces.

Our concern in this section is with friction, an important force in many aspects of everyday life. The oil in a car engine minimizes friction between moving parts, but without friction between the tires and the road we couldn't drive or turn the car. Air drag—the frictional force exerted by the air on a body moving through it—decreases automotive fuel economy but makes parachutes work. Without friction, nails would pull out, light bulbs would unscrew effortlessly, and riding a bicycle would be hopeless.

The sport of ice hockey depends crucially on having just the right amount of friction between a player's skates and the ice. If there were too much friction, the players would move much more slowly; if there were too little friction, they could hardly keep from falling over.

KINETIC AND STATIC FRICTION

Let's consider a body sliding across a surface. When you try to slide a heavy box of books across the floor, the box doesn't move at all unless you push with a certain minimum force. Then the box starts moving, and you can usually keep it moving with less force than you needed to get it started. If you take some of the books out, you need less force than before to get it started or keep it moving. What general statements can we make about this behavior?

First, when a body rests or slides on a surface, we can always represent the contact force exerted by the surface on the body in terms of components of force perpendicular and parallel to the surface. We call the perpendicular component vector the *normal force,* denoted by $\vec{n}$. (Recall that *normal* is a synonym for *perpendicular.*) The component vector parallel to the surface is the **friction force,** denoted by $\vec{f}$. By definition, $\vec{n}$ and $\vec{f}$ are always perpendicular to each other. We use script symbols for these quantities to emphasize their special role in representing the contact force. If the surface is frictionless, then the contact force has *only* a normal component, and $\vec{f}$ is zero. (Frictionless surfaces are an unattainable idealization, but we can approximate a surface as frictionless if the effects of friction are negligibly small.) The direction of the friction force is always such as to oppose relative motion of the two surfaces.

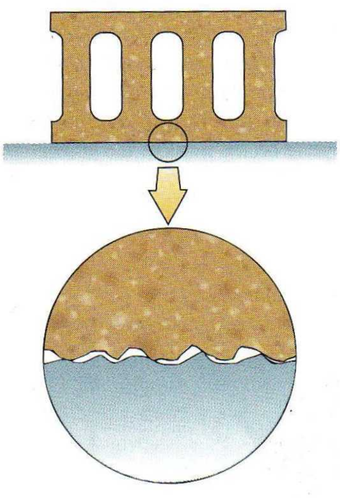
The kind of friction that acts when a body slides over a surface is called a **kinetic friction force** $\vec{f}_k$. The adjective "kinetic" and the subscript "k" remind us that the two surfaces are moving relative to each other. The *magnitude* of the kinetic friction force usually increases when the normal force increases. It takes more force to slide a box full of books across the floor than to slide the same box when it is empty. This principle is also used in automotive braking systems; the harder the brake pads are squeezed against the rotating brake disks, the greater the braking effect. In many cases the magnitude of the kinetic friction force f_k is found experimentally to be approximately *proportional* to the magnitude n of the normal force. In such cases we can write

$$f_k = \mu_k n \quad \text{(magnitude of kinetic friction force)}, \qquad (5\text{–}5)$$

where μ_k (pronounced "mu-sub-k") is a constant called the **coefficient of kinetic friction.** The more slippery the surface, the smaller the coefficient of friction. Because it is a quotient of two force magnitudes, μ_k is a pure number without units.

CAUTION ▶ Remember, the friction force and the normal force are always perpendicular. Equation (5–5) is *not* a vector equation, but a *scalar* relation between the *magnitudes* of the two perpendicular forces. ◀

Equation (5–5) is only an approximate representation of a complex phenomenon. On a microscopic level, friction and normal forces result from the intermolecular forces (fundamentally electrical in nature) between two rough surfaces at points where they come into contact (Fig. 5–15). The actual area of contact is usually much smaller than



5–15 The normal and friction forces arise from interactions between molecules at high points on the surfaces of the block and the floor.

the total surface area. As a box slides over the floor, bonds between the two surfaces form and break, and the total number of such bonds varies; hence the kinetic friction force is not perfectly constant. Smoothing the surfaces can actually increase friction, since more molecules are able to interact and bond; bringing two smooth surfaces of the same metal together can cause a "cold weld." Lubricating oils work because an oil film between two surfaces (such as the pistons and cylinder walls in a car engine) prevents them from coming into actual contact.

Table 5–1 shows a few representative values of μ_k. Although these values are given with two significant figures, they are only approximate, since friction forces can also depend on the *speed* of the body relative to the surface. We'll ignore this effect and assume that μ_k and f_k are independent of speed so that we can concentrate on the simplest cases. Table 5–1 also lists coefficients of *static* friction; we'll define these shortly.

TABLE 5–1
APPROXIMATE COEFFICIENTS OF FRICTION

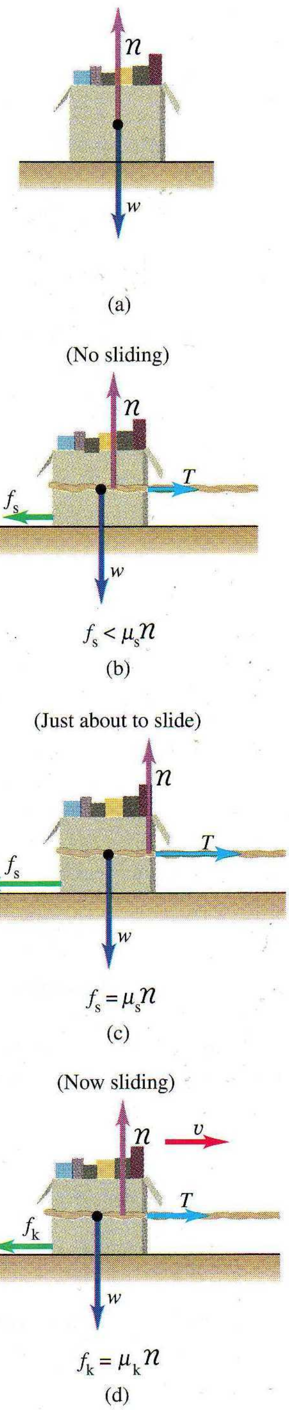
MATERIALS	STATIC, μ_s	KINETIC, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Copper on cast iron	1.05	0.29
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

Friction forces may also act when there is *no* relative motion. If you try to slide that box of books across the floor, the box may not move at all because the floor exerts an equal and opposite friction force on the box. This is called a **static friction force** $\vec{f}_s$. In Fig. 5–16a the box is at rest in equilibrium under the action of its weight $\vec{w}$ and the upward normal force $\vec{n}$, which is equal in magnitude to the weight and exerted on the box by the floor. Now we tie a rope to the box (Fig. 5–16b) and gradually increase the tension T in the rope. At first the box remains at rest because, as T increases, the force of static friction f_s also increases (staying equal in magnitude to T).

At some point, T becomes greater than the maximum static friction force f_s the surface can exert. Then the box "breaks loose" (the tension T is able to break the bonds between molecules in the surfaces of the box and floor) and starts to slide. Figure 5–16c is the force diagram when T is at this critical value. If T exceeds this value, the box is no longer in equilibrium. For a given pair of surfaces the maximum value of f_s depends on the normal force. Experiment shows that in many cases this maximum value, called $(f_s)_{max}$, is approximately *proportional* to n; we call the proportionality factor μ_s (pronounced "mu-sub-s") the **coefficient of static friction.** Some representative values of μ_s are shown in Table 5–1. In a particular situation, the actual force of static friction can have any magnitude between zero (when there is no other force parallel to the surface) and a maximum value given by $\mu_s n$. In symbols,

$$f_s \le \mu_s n \quad \text{(magnitude of static friction force).} \tag{5-6}$$

Like Eq. (5–5), this is a relation between magnitudes, *not* a vector relation. The



5–16 (a), (b), (c) When there is no relative motion of the surfaces, the magnitude of the static friction force f_s is less than or equal to $\mu_s n$. (d) When there is relative motion, the magnitude of the kinetic friction force f_k equals $\mu_k n$.

ActivPhysics

2.5
Truck Pulls Crate

2.6
Pushing a Crate

2.7
Pushing a Crate Up a Wall

2.8
Skier Goes Down a Slope

2.9
Skier and Rope Tow

2.10
Pushing a Crate Up an Incline

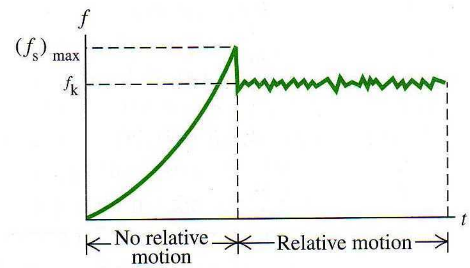
2.12
Truck Pulls Two Crates

equality sign holds only when the applied force T, parallel to the surface, has reached the critical value at which motion is about to start (Fig. 5–16c). When T is less than this value (Fig. 5–16b), the inequality sign holds. In that case we have to use the equilibrium conditions ($\Sigma \vec{F} = \mathbf{0}$) to find f_s. If there is no applied force ($T = 0$) as in Fig. 5–16a, then there is no static friction force either ($f_s = 0$).

As soon as sliding starts (Fig. 5–16d), the friction force usually *decreases;* it's easier to keep the box moving than to start it moving. Hence the coefficient of kinetic friction is usually *less* than the coefficient of static friction for any given pair of surfaces, as shown in Table 5–1. If we start with no applied force ($T = 0$) at time $t = 0$ and gradually increase the force, the friction force varies somewhat, as shown in Fig. 5–17.

In some situations the surfaces will alternately stick (static friction) and slip (kinetic friction). This is what causes the horrible squeak made by chalk held at the wrong angle while writing on the blackboard. Another stick-slip phenomenon is the squeaky noise your windshield-wiper blades make when the glass is nearly dry; still another is the outraged shriek of tires sliding on asphalt pavement. A more positive example is the motion of a violin bow against the string.

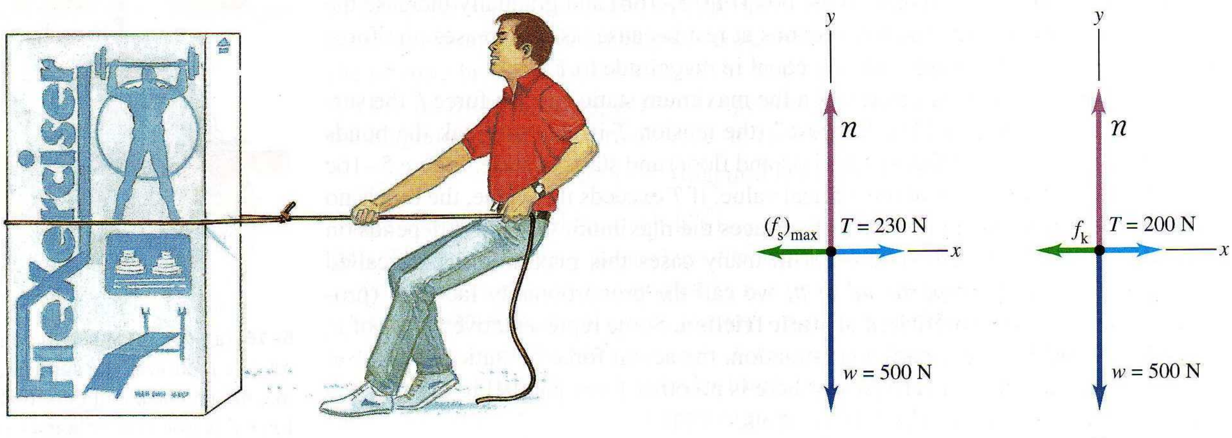
When a body slides on a layer of gas, friction can be made very small. In the linear air track used in physics laboratories, the gliders are supported on a layer of air. The frictional force is velocity-dependent, but at typical speeds the effective coefficient of friction is of the order of 0.001. A similar device is the air table, where the pucks are supported by an array of small air jets about 2 cm apart.



5–17 In response to an externally applied force, the friction force increases to $(f_s)_{max}$. Then the surfaces begin to slide across one another, and the frictional force drops back to a nearly constant value f_k. The kinetic friction force varies somewhat as intermolecular bonds form and break.

EXAMPLE 5–13

Friction in horizontal motion A delivery company has just unloaded a 500-N crate full of home exercise equipment on the sidewalk in front of your house (Fig. 5–18a). You find that to get it started moving toward your front door, you have to pull with a horizontal force of magnitude 230 N. Once it "breaks loose" and starts to move, you can keep it moving at constant velocity with only 200 N. What are the coefficients of static and kinetic friction?



5–18 (a) Pulling a crate with a horizontal force. (b) Free-body diagram for the crate as it starts to move. (c) Free-body diagram for the crate moving at constant velocity.

SOLUTION The state of rest and the state of motion with constant velocity are both equilibrium conditions, so we use Eqs. (5–2). An instant before the crate starts to move, the static friction force has its maximum possible value, $(f_s)_{max} = \mu_s n$. The appropriate force diagram is Fig. 5–18b. We find

$$\Sigma F_x = T + (-(f_s)_{max}) = 230\text{ N} - (f_s)_{max} = 0, \qquad (f_s)_{max} = 230\text{ N},$$

$$\Sigma F_y = n + (-w) = n - 500\text{ N} = 0, \qquad n = 500\text{ N},$$

$$(f_s)_{max} = \mu_s n \qquad \text{(motion about to start)},$$

$$\mu_s = \frac{(f_s)_{max}}{n} = \frac{230\text{ N}}{500\text{ N}} = 0.46.$$

After the crate starts to move, the forces are as shown in Fig. 5–18c, and we have

$$\Sigma F_x = T + (-f_k) = 200\text{ N} - f_k = 0, \qquad f_k = 200\text{ N},$$

$$\Sigma F_y = n + (-w) = n - 500\text{ N} = 0, \qquad n = 500\text{ N},$$

$$f_k = \mu_k n \qquad \text{(motion occurs)},$$

$$\mu_k = \frac{f_k}{n} = \frac{200\text{ N}}{500\text{ N}} = 0.40.$$

It's easier to keep the crate moving than to start it moving, and so the coefficient of kinetic friction is less than the coefficient of static friction.

EXAMPLE 5–14

In Example 5–13, what is the friction force if the crate is at rest on the surface and a horizontal force of 50 N is applied to it?

SOLUTION From the equilibrium conditions we have

$$\Sigma F_x = T + (-f_s) = 50\text{ N} - f_s = 0,$$

$$f_s = 50\text{ N}.$$

In this case, f_s is less than the maximum value $(f_s)_{max} = \mu_s n$. The frictional force can prevent motion for any horizontal applied force up to 230 N.

EXAMPLE 5–15

In Example 5–13, suppose you try to move your crate full of exercise equipment by tying a rope around it and pulling upward on the rope at an angle of 30° above the horizontal (Fig. 5–19a). How hard do you have to pull to keep the crate moving with constant velocity? Is this easier or harder than pulling horizontally? Assume that $w = 500$ N and $\mu_k = 0.40$.

SOLUTION Figure 5–19b is a free-body diagram showing the forces on the crate. The kinetic friction force f_k is still equal to $\mu_k n$, but now the normal force n is *not* equal in magnitude to the weight of the crate. The force exerted by the rope has an additional vertical component that tends to lift the crate off the floor.

The crate is in equilibrium, since its velocity is constant, so

$$\Sigma F_x = T\cos 30° + (-f_k) = T\cos 30° - 0.40n = 0,$$

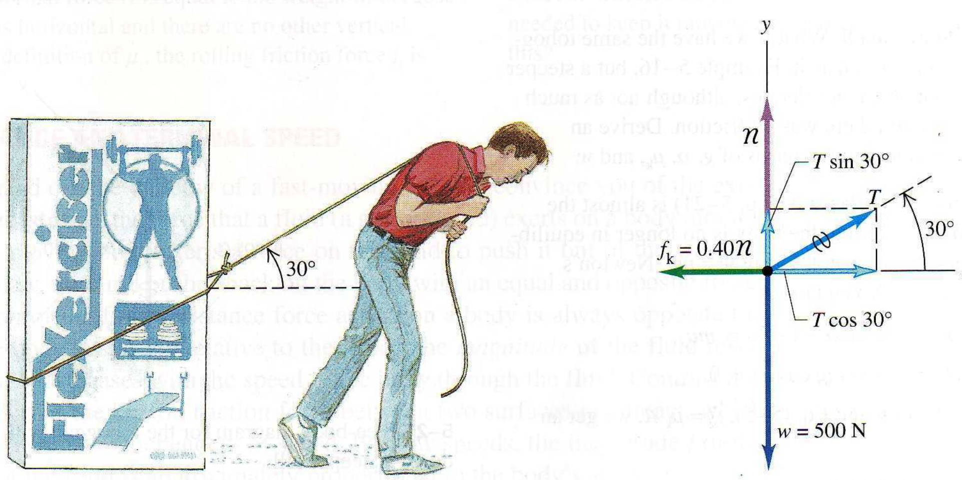
$$\Sigma F_y = T\sin 30° + n + (-500\text{ N}) = 0.$$

These are two simultaneous equations for the two unknown quantities T and n. To solve them, we can eliminate one unknown and solve for the other. There are many ways to do this; here is one way. Rearrange the second equation to the form

$$n = 500\text{ N} - T\sin 30°.$$

Substitute this expression for n back into the first equation:

$$T\cos 30° - 0.40(500\text{ N} - T\sin 30°) = 0.$$



(a) (b)

5–19 (a) Pulling a crate with a force applied at an upward angle. (b) Free-body diagram for the crate moving at constant velocity.

Finally, solve this equation for T, then substitute the result back into either of the original equations to obtain n. The results are

$$T = 188\ \text{N}, \qquad n = 406\ \text{N}.$$

Note that the normal force is *less* than the weight of the box ($w = 500$ N) because the vertical component of tension pulls upward on the crate. Despite this, the tension required is a little less than the 200-N force needed when you pulled horizontally in Example 5–13. Try pulling at 22°; you'll find that you need even less force (see Challenge Problem 5–111).

EXAMPLE 5–16

Toboggan ride with friction I Let's go back to the toboggan that we studied in Example 5–9 (Section 5–3). The wax has worn off, and there is now a coefficient of kinetic friction μ_k. The slope has just the right angle to make the toboggan slide with constant speed. Derive an expression for the slope angle in terms of w and μ_k.

SOLUTION Figure 5–20 shows the free-body diagram. The slope angle is α. The forces on the toboggan are its weight w and the normal and frictional components of the contact force exerted on it by the sloping surface. We take axes perpendicular and parallel to the surface and represent the weight in terms of its components in these two directions, as shown. The toboggan is in equilibrium because its velocity is constant, and the equilibrium conditions are

$$\Sigma F_x = w \sin\alpha + (-f_k) = w \sin\alpha - \mu_k n = 0,$$

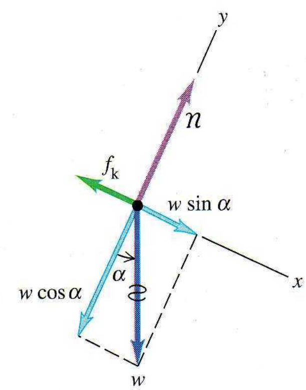
$$\Sigma F_y = n + (-w \cos\alpha) = 0.$$

Rearranging, we get

$$\mu_k n = w \sin\alpha, \qquad n = w \cos\alpha.$$

Just as in Example 5–9, the normal force n is *not* equal to the weight w. When we divide the first of these equations by the second, we find

$$\mu_k = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha.$$



5–20 Free-body diagram for the toboggan with friction.

The weight w doesn't appear in this expression. *Any* toboggan, regardless of its weight, slides down an incline with constant speed if the coefficient of kinetic friction equals the tangent of the slope angle of the incline. The steeper the slope, the greater the coefficient of friction has to be for the toboggan to slide with constant velocity. This is just what we should expect.

EXAMPLE 5–17

Toboggan ride with friction II What if we have the same toboggan and coefficient of friction as in Example 5–16, but a steeper hill? This time the toboggan accelerates, although not as much as in Example 5–9, when there was no friction. Derive an expression for the acceleration in terms of g, α, μ_k, and w.

SOLUTION The free-body diagram (Fig. 5–21) is almost the same as for Example 5–16, but the body is no longer in equilibrium; a_y is still zero, but a_x is not. Using $w = mg$, Newton's second law gives us the two equations

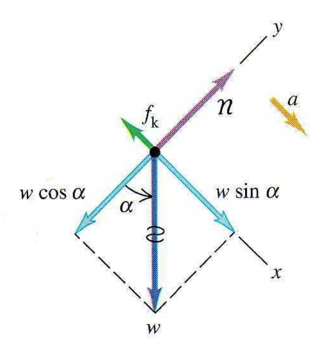
$$\Sigma F_x = mg \sin\alpha + (-f_k) = ma_x,$$

$$\Sigma F_y = n + (-mg \cos\alpha) = 0.$$

From the second equation and Eq. (5–5), $f_k = \mu_k n$, we get an expression for f_k:

$$n = mg \cos\alpha,$$

$$f_k = \mu_k n = \mu_k mg \cos\alpha.$$



5–21 Free-body diagram for the toboggan with friction, going down a steeper hill.

We substitute this back into the x-component equation. The result is

$$mg \sin \alpha + (-\mu_k mg \cos \alpha) = ma_x,$$

$$a_x = g(\sin \alpha - \mu_k \cos \alpha).$$

Does this result make sense? Here are some special cases we can check. First, if the hill is *vertical,* $\alpha = 90°$; then $\sin \alpha = 1$, $\cos \alpha = 0$, and $a_x = g$. This is free fall, just what we would expect. Second, on a hill at angle α with *no* friction, $\mu_k = 0$. Then $a_x = g \sin \alpha$. The situation is the same as in Example 5–9, and we get the same result; that's encouraging! Next, suppose that there is just enough friction to make the toboggan move with constant velocity. In that case, $a_x = 0$, and our result gives

$$\sin \alpha = \mu_k \cos \alpha \quad \text{and} \quad \mu_k = \tan \alpha.$$

This agrees with our result from Example 5–16; good! Finally, note that there may be so much friction that $\mu_k \cos \alpha$ is actually greater than $\sin \alpha$. In that case, a_x is negative; if we give the toboggan an initial downhill push to start it moving, it will slow down and eventually stop.

We have pretty much beaten the toboggan problem to death, but there is an important lesson to be learned. We started out with a simple problem and then extended it to more and more general situations. Our most general result, found in this example, includes *all* the previous ones as special cases, and that's a nice, neat package! Don't memorize this package; it is useful only for this one set of problems. But do try to understand how we obtained it and what it means.

One final variation that you may want to try out is the case in which we give the toboggan an initial push *up* the hill. The direction of the friction force is now reversed, so the acceleration is different from the downhill value. It turns out that the expression for a_x is the same as for downhill motion except that the minus sign becomes plus. Can you prove this?

ROLLING FRICTION

It's a lot easier to move a loaded filing cabinet across a horizontal floor by using a cart with wheels than to slide it. How much easier? We can define a **coefficient of rolling friction** μ_r, which is the horizontal force needed for constant speed on a flat surface divided by the upward normal force exerted by the surface. Transportation engineers call μ_r the *tractive resistance.* Typical values of μ_r are 0.002 to 0.003 for steel wheels on steel rails and 0.01 to 0.02 for rubber tires on concrete. These values show one reason why railroad trains are in general much more fuel-efficient than highway trucks.

EXAMPLE 5–18

Motion with rolling friction A typical car weighs about 12,000 N (about 2700 lb). If the coefficient of rolling friction is $\mu_r = 0.010$, what horizontal force must you apply to push the car at constant speed on a level road? Neglect air resistance.

SOLUTION The normal force n is equal to the weight w, because the road surface is horizontal and there are no other vertical forces. From the definition of μ_r, the rolling friction force f_r is

$$f_r = \mu_r n = (0.010)(12{,}000 \text{ N}) = 120 \text{ N} \quad \text{(about 27 lb).}$$

From Newton's first law, a forward force with this magnitude is needed to keep the car moving with constant speed.

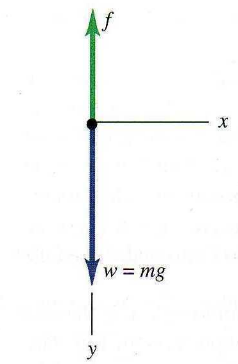
We invite you to apply this analysis to your crate of exercise equipment (Example 5–13). If the delivery company brings it on a rubber-wheeled dolly with $\mu_r = 0.02$, only a 10-N force is needed to keep it moving at constant velocity. Can you verify this?

FLUID RESISTANCE AND TERMINAL SPEED

Sticking your hand out the window of a fast-moving car will convince you of the existence of **fluid resistance,** the force that a fluid (a gas or liquid) exerts on a body moving through it. The moving body exerts a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the body with an equal and opposite force.

The *direction* of the fluid resistance force acting on a body is always opposite the direction of the body's velocity relative to the fluid. The *magnitude* of the fluid resistance force usually increases with the speed of the body through the fluid. Contrast this behavior with that of the kinetic friction force between two surfaces in contact, which we can usually regard as independent of speed. For low speeds, the magnitude f of the resisting force of the fluid is approximately proportional to the body's speed v:

$$f = kv \quad \text{(fluid resistance at low speed),} \tag{5–7}$$



5–22 Free-body diagram for a body falling through a fluid.

where k is a proportionality constant that depends on the shape and size of the body and the properties of the fluid. In motion through air at the speed of a tossed tennis ball or faster, the resisting force is approximately proportional to v^2 rather than to v. It is then called **air drag** or simply *drag*. Airplanes, falling raindrops, and cars moving at high speed all experience air drag. In this case we replace Eq. (5–7) by

$$f = Dv^2 \qquad \text{(fluid resistance at high speed).} \qquad (5\text{–}8)$$

Because of the v^2 dependence, air drag increases rapidly with increasing speed. The air drag on a typical car is negligible at low speeds but comparable to or greater than the force of rolling resistance at highway speeds. The value of D depends on the shape and size of the body and on the density of the air.

We invite you to show that the units of the constant k in Eq. (5–7) are N · s/m or kg/s and that the units of the constant D in Eq. (5–8) are N · s^2/m^2 or kg/m.

Because of the effects of fluid resistance, an object falling in a fluid will *not* have a constant acceleration. To describe its motion, we can't use the constant-acceleration relationships from Chapter 2; instead, we have to start over, using Newton's second law. Let's consider the following situation. You release a rock at the surface of a deep pond, and it falls to the bottom. The fluid resistance force in this situation is given by Eq. (5–7). What are the acceleration, velocity, and position of the rock as functions of time?

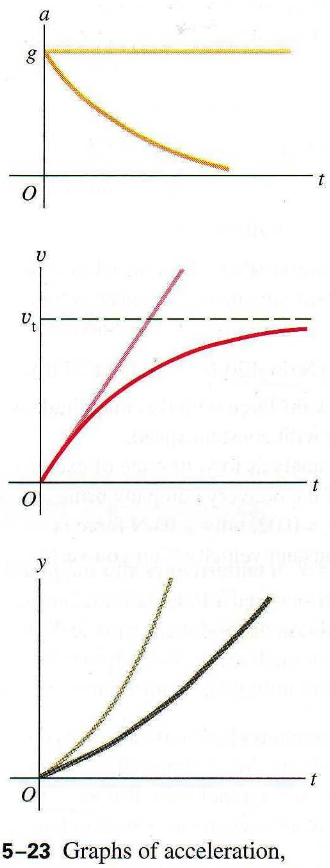
The free-body diagram is shown in Fig. 5–22. We take the positive direction to be downward and neglect any force associated with buoyancy in the water. There are no x-components, and Newton's second law gives

$$\Sigma F_y = mg + (-kv) = ma.$$

When the rock first starts to move, $v = 0$, the resisting force is zero, and the initial acceleration is $a = g$. As its speed increases, the resisting force also increases until finally it is equal in magnitude to the weight. At this time, $mg - kv = 0$, the acceleration becomes zero, and there is no further increase in speed. The final speed v_t, called the **terminal speed,** is given by $mg - kv_t = 0$, or

$$v_t = \frac{mg}{k} \qquad \text{(terminal speed, fluid resistance } f = kv\text{).} \qquad (5\text{–}9)$$

Figure 5–23 shows how the acceleration, velocity, and position vary with time. As time goes by, the acceleration approaches zero, and the velocity approaches v_t (remember that we chose the positive y-direction to be down). The slope of the graph of y versus t becomes constant as the velocity becomes constant.



5–23 Graphs of acceleration, velocity, and position versus time for a body falling with fluid resistance proportional to v, shown as dark color curves. The light color curves show the corresponding relations if there is *no* fluid resistance.

ActivPhysics
2.1.2
Skydiver

In deriving the terminal speed in Eq. (5–9), we assumed that the fluid resistance force was proportional to the speed. For an object falling through the air at high speeds, so that the fluid resistance is proportional to v^2 as in Eq. (5–8), we invite you to show that the terminal speed v_t is given by

$$v_t = \sqrt{\frac{mg}{D}} \quad \text{(terminal speed, fluid resistance } f = Dv^2\text{).} \qquad (5\text{–}13)$$

This expression for terminal speed explains the observation that heavy objects in air tend to fall faster than light objects. Two objects with the same physical size but different mass (say, a table-tennis ball and a lead ball with the same radius) have the same value of D but different values of m. The more massive object has a larger terminal speed and falls faster. The same idea explains why a sheet of paper falls faster if you first crumple it into a ball; the mass m is the same, but the smaller size makes D smaller (less air drag for a given speed) and v_t larger.

By changing the positions of their arms and legs while falling, skydivers can change the value of the constant D in Eq. (5–8) and hence adjust the terminal speed of their fall (Eq. (5–13)).

EXAMPLE 5–19

Terminal speed of a sky diver For a human body falling through air in a spread-eagle position, the numerical value of the constant D in Eq. (5–8) is about 0.25 kg/m. For an 80-kg sky diver the terminal velocity is, from Eq. (5–13),

$$v_t = \sqrt{\frac{mg}{D}} = \sqrt{\frac{(80\ \text{kg})(9.8\ \text{m/s}^2)}{0.25\ \text{kg/m}}}$$

$$= 56\ \text{m/s} \qquad \text{(about 200 km/h, or 125 mi/h).}$$

When the sky diver deploys the parachute, the value of D increases greatly, and the terminal speed of the sky diver and parachute is (thankfully) much less than 56 m/s.