Significant Figures

When you write down a number that has some uncertainty, there are only a few decimal digits of that number you can trust, the rest could be wrong. The *significant figures* are the digits you can trust (although the right-most significant figure could be off by one); the rest are *insignificant*. The zeroes that are there *only* to establish where the decimal point is are also considered insignificant.

Examples:

- **1234**, all four figures are significant. Implied precision 1234 ± 1.
- **91.3**, all three figures are significant. Implied precision 91.3 ± 0.1.
- **37**00, only 2 figures are significant, the zeroes are not. Implied precision 3700 ± 100.
- 0.00**145**. only 3 figures are significant, the zeroes are not. Implied precision 0.00145 ± 0.00001.
- **6.3800**, all 5 figures are significant, including the zeroes. Implied precision 6.3800 ± 0.0001.
- 0.0**340**, three significant figures, including the zero on the right, but the zeroes on the left are not significant. Implied precision 0.0340 ± 0.0001.
- **201**$\underline{\mathbf{0}}$, four significant figures, including the underlined zero. Implied precision 2010 ± 1. Note: underlining a zero on the right indicates that we can trust it. Without underlining, the implied accuracy of 2010 is 2010 ± 10 and there are only three significant figures.

⋆ To avoid ambiguity, use scientific notations: $\mathbf{2.010} \cdot 10^3$ has four significant digits while $\mathbf{2.01} \cdot 10^3$ has only three.

Rule: *Don't write down more significant figures than you are sure of.*
For example, what's the distance between Austin and San Antonio? If you don't know where exactly in Austin you start and where exactly in San Antonio you end up, your accuracy could not possibly be better than give-or-take a few miles, so you should not write the distance as 79.3 miles or even as 79 miles but round it off to 80 miles. And if you convert it

to kilometers as 80 miles $\times$ 1.61 km/mile = 129 km, that answer would be too precise and you should round it off to 130 km.

Exception: At intermediate stages of a calculation, keep all the figures you get from your calculator. When you finish the calculation, then round off your answer to the appropriate number of significant figures.

Significant Figures in Calculations

When you apply complicated formulae to numbers with uncertainties, figuring out the uncertainty of the result could be quite complicated. But you can get a quick and dirty estimate by simply counting the significant figures.

When you multiply two or more numbers, count the significant figures of all the factors. The product should have as many significant figures as the least accurate factor. For example, consider the area $A = ab$ of a rectangle with sides $a = 12.52$ m by $b = 4.1$ m. Here one factor a has 4 significant figures but the other factor b has only 2, so the product $A = ab = 51.332$ m^2 should be rounded to 2 significant figures only, $A = 51$ m^2.

Same rule works for division: count the significant figures of the numerator and the denominator, and use the lesser of the two for the ratio.

When you add or subtract numbers, instead of counting the significant figures, note the decimal position of the last (rightmost) significant figure of each number. The last significant figure of the sum or difference follows from the least accurate term. For example, if you first walk 3500 meters and then another 214 meters, then the total distance is

$$s = \mathbf{35}00 \text{ m} + \mathbf{214} \text{ m} = \mathbf{37}14 \text{ m} \approx \mathbf{37}00 \text{ m}$$

where only the thousands and the hundreds of meters are significant. The single meters and the tens of meters are significant in the second term but not the first, so they are not significant in the sum.