Problem #**1**:

By definition,

$$\text{average speed} \;=\; \frac{\text{net distance along the path of motion}}{\text{time}}\,. \tag{1}$$

Note: the net distance along the path is a scalar quantity — its direction is not important — so the average speed is also a scalar.

The student in question walks along 3 sides of a square mile, so the net distance *along his path of motion* is 3 miles. The time this motion took was 1 hour, so the average speed of the student was 3 miles per hour.

On the other hand, the average velocity is a vector quantity defined as

$$\text{average velocity} \;=\; \frac{\text{net displacement vector}}{\text{time}}\,. \tag{2}$$

The net displacement vector of the student is

$$\vec{D}_{\text{net}} \;=\; (1\text{ mile South}) \;+\; (1\text{ mile West}) \;+\; (1\text{ mile North}) \;=\; (1\text{ mile West}) \tag{3}$$

since the Southward and Northward displacements cancel each other. Consequently, the student's average velocity vector is 1 mile per hour in the direction due West.

Problem #**2**:

(a) Since the car has a constant acceleration a, its velocity changes with time according to a very simple formula

$$v(t) \;=\; v_0 \;+\; at \tag{4}$$

where v_0 is the initial velocity. Note that the acceleration a is negative, so the velocity decreases with time.

Given a and the final velocity v, we may find the deceleration time t by simply solving the equation (4) for t:

$$t \;=\; \frac{v-v_0}{a} \;=\; \frac{(12\text{ m/s})\;-\;(32\text{ m/s})}{-2.5\text{ m/s}^2} \;=\; \frac{-2\underline{0}\text{ m/s}}{-2.5\text{ m/s}^2} \;=\; 8.0\text{ s.} \tag{5}$$

(b) In class, I showed that when a body has constant acceleration (or deceleration), its average velocity is the average between the initial and the final velocity:

$$v_{\rm avg} = \frac{v_{\rm init} + v_{\rm fin}}{2}. \tag{6}$$

Note that this formula applies only when the acceleration is constant!

For the car in question, eq. (6) gives its average velocity during the period it was decelerating:

$$v_{\rm avg} = \frac{v_0 + v}{2} = \frac{(32\ {\rm m/s})\ +\ (12\ {\rm m/s})}{2} = 22\ {\rm m/s}. \tag{7}$$

Consequently, the distance covered by the car during this time is

$$D = t \times v_{\rm avg} = 8.0\ {\rm s} \times 22\ {\rm m/s} = 176\ {\rm m} \approx 180\ {\rm m}, \tag{8}$$

or about 200 yards (600 feet).

Alternative solution: Use the equation of motion at constant acceleration,

$$D = x - x_0 = v_0 \times t + \frac{a}{2} \times t^2. \tag{9}$$

For the time t we found in part (a), this formula gives us

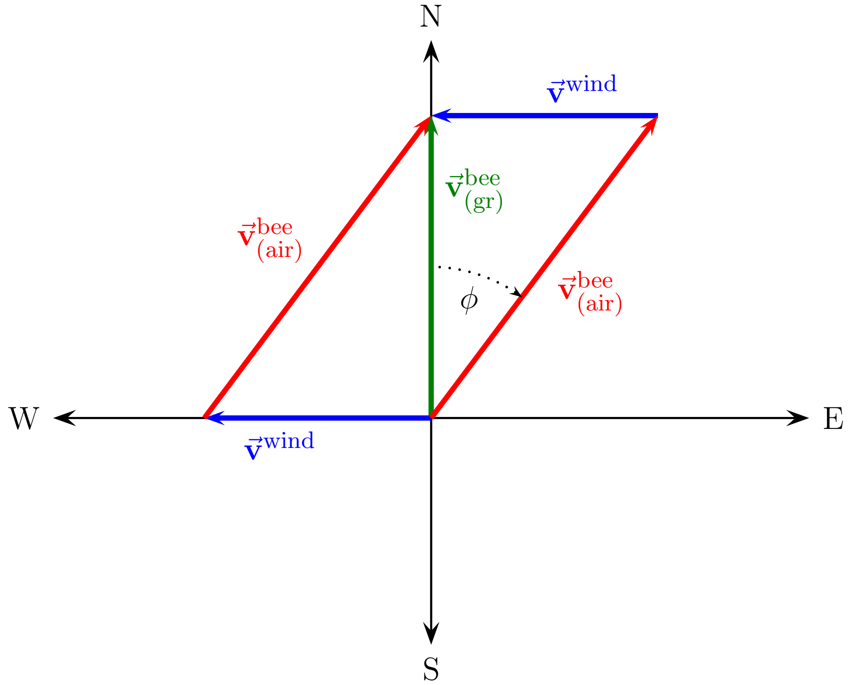
$$D = (32\,{\rm m/s})\times(8.0\,{\rm s}) + \frac{(-2.5\ {\rm m/s^2})}{2}\times(8.0\,{\rm s})^2 = 256\,{\rm m} - 8\underline{0}\,{\rm m} = 176\,{\rm m} \approx 180\,{\rm m}. \tag{10}$$

Problem #**3**:

The velocity vectors of the bee relative to the ground and relative to the moving air differ by the wind velocity vector,

$$\vec{\mathbf{v}}^{\rm bee}_{\rm (gr)} = \vec{\mathbf{v}}^{\rm bee}_{\rm (air)} + \vec{\mathbf{v}}^{\rm wind}, \qquad \vec{\mathbf{v}}^{\rm bee}_{\rm (air)} = \vec{\mathbf{v}}^{\rm bee}_{\rm (gr)} - \vec{\mathbf{v}}^{\rm wind}. \tag{11}$$

We know the direction and magnitude of the wind velocity vector — 9 MPH due West — and also direction and magnitude of bee's velocity vector relative to the ground — 12 MPH due North. To find the bee's velocity vector relative to the air, all we need is to evaluate the vector difference in the second eq. (11). Graphically,



In (N,E) components,

$$\left(v^{\rm bee}_{\rm (gr)}\right)_N = 12\ {\rm MPH},\quad \left(v^{\rm bee}_{\rm (gr)}\right)_E = 0,\quad \left(v^{\rm wind}\right)_N = 0,\quad \left(v^{\rm wind}\right)_E = -9\ {\rm MPH}, \tag{12}$$

hence

$$\begin{aligned} \left(v^{\rm bee}_{\rm (air)}\right)_N &= \left(v^{\rm bee}_{\rm (gr)}\right)_N - \left(v^{\rm wind}\right)_N = 12\ {\rm MPH} - 0\ {\rm MPH} = +12\ {\rm MPH},\\ \left(v^{\rm bee}_{\rm (air)}\right)_E &= \left(v^{\rm bee}_{\rm (gr)}\right)_E - \left(v^{\rm wind}\right)_E = 0\ {\rm MPH} - (-9\ {\rm MPH}) = +9\ {\rm MPH}. \end{aligned} \tag{13}$$

Now let's convert these components of the bee's velocity vector relative to the air into magnitude and direction. The magnitude — *i.e.*, the bee's *airspeed* — obtains from the Pythagoras formula

$$v^{\rm bee}_{\rm (air)} = \sqrt{\left(v^{\rm bee}_{\rm (air)}\right)_N^2 + \left(v^{\rm bee}_{\rm (air)}\right)_E^2} = \sqrt{(12\ {\rm MPH})^2 + (9\ {\rm MPH})^2} = 15\ {\rm MPH}. \tag{14}$$

As to the direction of the $\vec{\mathbf{v}}^{\rm bee}_{\rm (air)}$ — *i.e.*, the bee's *heading*, — we have

$$\tan\phi = \frac{\left(\vec{\mathbf{v}}^{\rm bee}_{\rm (air)}\right)_E}{\left(\vec{\mathbf{v}}^{\rm bee}_{\rm (air)}\right)_N} = \frac{9\ {\rm MPH}}{12\ {\rm MPH}} = 0.75, \tag{15}$$

hence

$$\phi \;=\; \arctan(0.75) \quad \bigl(\text{modulo } 180^\circ\bigr) \;=\; 37^\circ \text{ or } 217^\circ. \tag{16}$$

Since the North and East components of the $\vec{\mathbf{v}}^{\text{bee}}_{(\text{air})}$ are both positive, the correct answer is 37°. Thus, the bee in questions has *heading* 37° (clockwise from North).

Problem #**4**:

Once the car exits the surviving part of the bridge, it flies through the air like a projectile until it hits the water. The vertical and the horizontal motion of a projectile are independent:

$$x(t) \;=\; v_{0x}t, \qquad y(t) \;=\; y_0 \;+\; v_{0y}t \;-\; \tfrac{1}{2}gt^2, \tag{17}$$

where $\vec{\mathbf{v}}_0 = (v_{0x}, v_{0y})$ is the initial velocity vector of the car at the moment it went off the bridge. The direction of this vector is parallel to the bridge, which is presumably horizontal. Therefore, $v_{0x} = v_0$ is the car's speed when it went off the bridge while $v_{0y} = 0$, hence

$$x(t) \;=\; v_0 t, \tag{18}$$

$$y(t) \;=\; y_0 \;-\; \tfrac{1}{2}gt^2. \tag{19}$$

(a) Note that the vertical motion of the car does not depend on its initial speed but only of the time of its flight. Consequently, we may obtain this time of flight t from the net vertical displacement of the car from the bridge $y_0 = 16$ m to the water surface $y = 0$. Thus, solving eq. (19) for the time t, we find

$$y_0 - \tfrac{1}{2}gt^2 \;=\; y \;=\; 0 \quad\Longrightarrow\quad \frac{g}{2}\times t^2 \;=\; y_0 \quad\Longrightarrow\quad t^2 \;=\; \frac{2y_0}{g} \quad\Longrightarrow\quad t \;=\; \sqrt{\frac{2y_0}{g}}. \tag{20}$$

For the bridge in question,

$$t \;=\; \sqrt{\frac{2\times 16\text{ m}}{9.8\text{ m/s}^2}} \;=\; \sqrt{3.27\text{ s}^2} \;\approx\; 1.8\text{ s}. \tag{21}$$

(b) Eq. (18) relates the horizontal displacement of the falling car to its initial speed v_0 and the time of flight t. Given the known horizontal displacement $x = 36$ m and the flight time $t = 1.8$ s found in part (a), the initial speed of the car follows as

$$v_0 = \frac{x}{t} = \frac{36\ \text{m}}{1.8\ \text{s}} = 20\ \text{m/s}, \tag{22}$$

or about 45 miles per hour.

Problem #**5**:

The Second Law of Newton relates the acceleration of the body with the *net force* acting on it,

$$m\vec{\mathbf{a}} = \vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots. \tag{23}$$

Note the vector sum here: When adding forces, it's important to know not only their magnitudes but also their directions.

The boat in question is subject to two horizontal forces: the propeller force $\vec{\mathbf{F}}_{\text{prop}} = +450$ N pushing the boat forward, and the water resistance force $\vec{\mathbf{F}}_{\text{res}} = -330$ N pushing the boat back. The $\pm$ signs here indicate the directions of the two forces. In one dimension — forward or backward — the vector sum of two forces is just the algebraic sum of the two signed numbers. Thus, the net force acting on the boat is

$$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_{\text{prop}} + \vec{\mathbf{F}}_{\text{res}} = (450\ \text{N}) + (-330\ \text{N}) = +120\ \text{N}, \tag{24}$$

where the + sign indicates the forward direction.

Given the net force on the boat, its acceleration follows from the Second Law,

$$\vec{\mathbf{a}} = \frac{1}{m}\vec{\mathbf{F}}_{\text{net}} = \frac{+120\ \text{N}/}{240\ \text{kg}} = +0.50\ \text{m/s}^2. \tag{25}$$

In other words, the boat accelerates forward at the rate $a = 0.50\ rmm/s^2$.