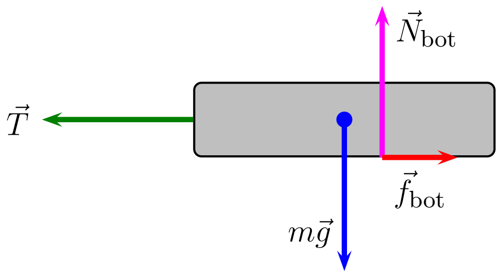
Problem #**1**:

(a) Forces on the top block:

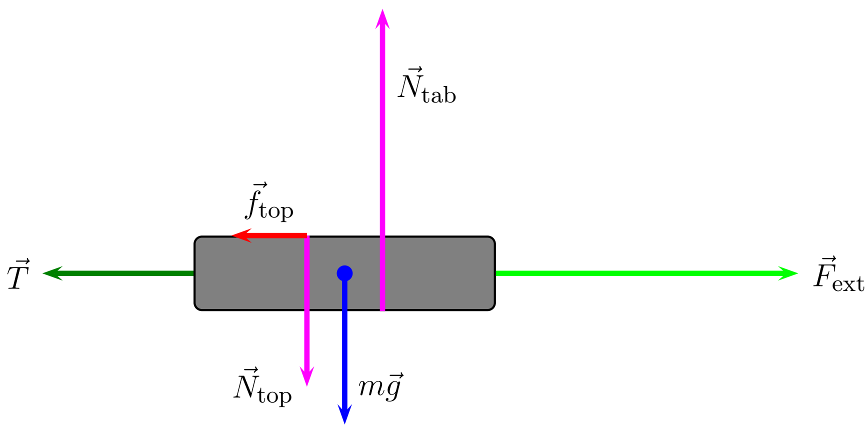


$\vec{T}$ — string force,

$\vec{N}_{\rm bot}$ — normal force from the bottom block,

$\vec{f}_{\rm bot}$ — kinetic friction force from the bottom block. (1)

Forces on the top block



(2)

Here $\vec{T}$ is the string force on the bottom block (same as on the top block), $\vec{F}_{\rm ext}$ is the external force pulling the block to the right, $\vec{N}_{\rm top}$ is the normal force from the top block, $\vec{N}_{\rm tab}$ is the normal force from the table, and $\vec{f}_{\rm top}$ is the kinetic friction force from the top block.

According to Newton's Third Law, $\vec{N}_{\rm top} = -\vec{N}_{\rm bot}$ and $\vec{f}_{\rm top} = -\vec{f}_{\rm bot}$. In particular, the kinetic friction between the blocks acts on both of them with forces of equal magnitudes $f = 3.0$ N but in opposite directions: it pulls the top block to the right (against its motion relative to the bottom block) while pulling the bottom block to the left (against its motion relative to the top block).

(b) For each block, Newton's Second Law gives us $m\vec{a} = \vec{F}_{\rm net}$(on that block), or in components

$$ma_x \;=\; F_x^{\rm net}(\text{on that block}), \qquad ma_y \;=\; F_y^{\rm net}(\text{on that block}). \tag{3}$$

Specifically, in coordinates where the x axis points right and the y axis points up,

$$ma_x^{\rm top} \;=\; F_x^{\rm net\,on\,top} \;=\; -T \;+\; f, \tag{4.1}$$
$$ma_y^{\rm top} \;=\; F_y^{\rm net\,on\,top} \;=\; -mg \;+\; N_{\rm bot}\,, \tag{4.2}$$
$$ma_x^{\rm bot} \;=\; F_x^{\rm net\,on\,bot} \;=\; -T \;-\; f \;+\; F_{\rm ext}\,, \tag{4.3}$$
$$ma_y^{\rm bot} \;=\; F_y^{\rm net\,on\,bot} \;=\; -mg \;-\; N_{\rm top} \;+\; N_{\rm floor}\,. \tag{4.4}$$

(c) We know that the top block accelerates to the left at the rate $a = 2.0\ \mathrm{m/s^2}$, thus $a_x^{\rm top} = -2.0\ \mathrm{m/s^2}$. (Note the negative sigh!) Equation (4.1) relates this horizontal acceleration to the string tension T and the kinetic friction force f between the two blocks. Since we know the friction force $f = 3.0$ N, the string tension T obtains as

$$T \;=\; f \;-\; ma_x^{\rm top} \;=\; (3.0\ \mathrm{N}) \;-\; (2.0\ \mathrm{kg}) \times (-2.0\ \mathrm{m/s^2}) \;=\; 3.0\ \mathrm{N} + 4.0\ \mathrm{N} \;=\; 7.0\ \mathrm{N}. \tag{5}$$

(d) The bottom block accelerates to the right at the rate $a = 2.0\ \mathrm{m/s^2}$, thus $a_x^{\rm bot} = +2.0\ \mathrm{m/s^2}$. Equation (4.3) relates this acceleration to the horizontal forces acting on the bottom block, namely the external force, the string tension, and the kinetic friction. Since we are given the friction force $f = 3.0$ N and we have found the string tension $T = 7.0$ N in part (c), we may solve eq. (4.3) for the external force:

$$\begin{aligned} F_{\rm ext} \;&=\; T \;+\; f \;+ ma_x^{\rm bot} \;=\; (7.0\ \mathrm{N}) \;+\; (3.0\ \mathrm{N}) \;+\; (2.0\ \mathrm{kg}) \times (+2.0\ \mathrm{m/s^2}) \\ &=\; 7.0\ N \;+\; 3.0\ N \;+\; 4.0\ N \;=\; 14.0\ N. \end{aligned} \tag{6}$$

Problem #**2**:

For a planet moving in a circle around a star, the gravitational force from the star provides for the centripetal acceleration of the planet

$$F_{\rm grav} \;=\; \frac{GM_*m_p}{R^2} \;=\; m_p\times a_c \;=\; m_p\times\frac{v^2}{R} \tag{7}$$

where v is the orbital speed of the planet. Solving eq. (7) for the speed v, we obtain

$$v^2 \;=\; \frac{GM_*}{R} \quad\Longrightarrow\quad v \;=\; \sqrt{\frac{GM_*}{R}}\,. \tag{8}$$

The length of a circular orbit is $2\pi R$, so the time the planet takes to make one full circle around the star is

$$T \;=\; \frac{2\pi R}{v} \;=\; 2\pi\sqrt{\frac{R^3}{GM_*}}\,. \tag{9}$$

Similar formula applies for the Earth's orbit around the Sun: the orbital period T_E — *i.e.*, the Earth's year — is related to the radius $R_E = 1$ au of the Earth orbit and the Sun's mass $M_\odot$ as

$$T_E \;=\; 2\pi\sqrt{\frac{R_E^3}{GM_\odot}}\,. \tag{10}$$

The problem gives us the orbital period of the other planet in units of Earth's years, so let's compare eqs. (9) and (10):

$$\frac{T}{T_E} \;=\; \sqrt{\frac{R^3}{GM_*}}\Bigg/\sqrt{\frac{R_E^3}{GM_\odot}} \tag{11}$$

(the factors of 2π cancel out). Taking the squares of both sides of this equation, we obtain

$$\left(\frac{T}{T_E}\right)^2 \;=\; \frac{R^3}{GM_*}\Bigg/\frac{R_E^3}{GM_\odot} \;=\; \frac{R^3\times GM_\odot}{R_E^3\times GM_*} \;=\; \left(\frac{R}{R_E}\right)^3\Bigg/\left(\frac{M_*}{M_\odot}\right) \tag{12}$$

(the Newton's constant G cancels out). On the right hand side, $R_E = 1$ au by definition of the astronomical unit, thus

$$\frac{R}{R_E} = R[\text{in units of au}]. \tag{13}$$

Likewise, on the left hand side

$$\frac{T}{T_E} = T[\text{in units of Earth's years}]. \tag{14}$$

Altogether, eq. (12)relates the planet's year T in units of Earth years to orbital radius R in units of au to the starts mass in units of Sun's mass,

$$\big(T[\text{in Earth's years}]\big)^2 = \frac{\big(R[\text{in au}]\big)^3}{M_{\text{star}}[\text{in Sun's masses}]}\,. \tag{15}$$

The problem gives us $T = 4.0$ Earth's years and $R = 2.0$ au. Plugging these number into eq. (15), we obtain

$$(4.0)^2 = \frac{(2.0)^3}{M_{\text{star}}[\text{in Sun's masses}]} \tag{16}$$

and hence

$$M_{\text{star}}[\text{in Sun's masses}] = \frac{(2.0)^3}{(4.0)^2} = \frac{8.0}{16} = 0.5. \tag{17}$$

In other words, the star in question has mass $M_{\text{star}} = 0.5\,M_{\text{Sun}}$.

Problem #**3**:

Let me give two solutions to this problem: (a) a complete solution using the work-energy theorem, and (b) direct calculation of the work under similifying assumptions.

(a) Let's use the work-energy theorem: The kinetic + potential energy of the student and her bike

$$E = K + U = \tfrac{1}{2}mv^2 + mgy \tag{18}$$

changes by the net work of all the forces — except the gravity force mg whose work is already accounted by the potential energy,

$$\Delta E = W^{\text{net}}. \tag{19}$$

Since the student's move from one building to another begins with $v_{\text{initial}} = 0$ and ends with $v_{\text{final}} = 0$, the net change of kinetic energy is zero

$$\Delta K = \tfrac{1}{2}mv_{\text{final}}^2 - \tfrac{1}{2}mv_{\text{initial}}^2 = 0 - 0 = 0. \tag{20}$$

At the same time, her destination is 10 meters higher than her starting point, so the potential energy increases by

$$\Delta U = mg \times \Delta y = 700 \text{ N} \times 10 \text{ m} = 7000 \text{ J}, \tag{21}$$

so the net kinetic + potential energy increases by

$$\Delta E = \Delta K + \Delta U = 0 + 7000 \text{ J} = 7000 \text{ J}. \tag{22}$$

Consequently, the net work of all forces except gravity must total 7000 Joules.

Besides gravity, the forces acting on the student + bike system include:

1. Normal force from the ground. Since this force is always perpendicular to the bike's motion, its work is zero.
2. The forward force on the bike. Its immediate origin is the static friction force between the wheels and the ground, but ultimately the forward force is due to the student pushing the bike's pedals. Indeed, the bike is basically a machine that converts the rider's force on the pedals into the forward force on the bike+rider system. So assuming perfect efficiency of this machine, the work of the forward force precisely equals to the student's work.
3. Air drag resisting the bike's motion. This force does negative work, but for a slow ride the drag force is small and its negative work may be neglected.
4. Kinetic friction force during breaking. The work of this force is always negative, but by assumptions of the problem this negative work is small enough to be negligible.

Altogether, the net work of all the forces except gravity amounts to

$$\begin{aligned} W^{\text{net}} &= W(N) \;+\; W(\text{student}) \;+\; W(\text{air drag}) \;+\; W(\text{break}) \\ &= 0 \;+\; W(\text{student}) \;-\; \text{negligible} \;-\; \text{negligible} \\ &\approx W(\text{student}). \end{aligned} \tag{23}$$

In other words, net work of all the forces except gravity is approximately just the work of the student riding the bike.

By the work-energy theorem, this net work equals to the change of the net kinetic + potential energy, $W^{\text{net}} = \Delta E = 7000$ J. Consequently, the student's mechanical work during her ride was 7000 Joules.

(b) The alternative solution calculates the student's work directly, but needs additional simplifying assumtions: (1) The road from one building to another has a constant incline θ to the horizontal,

$$\sin\theta \;=\; \frac{\Delta y = 10\text{ m}}{L = 1000\text{ m}} \;=\; 0.01 \;=\; 1\%. \tag{24}$$

(2) The student rides at constant speed; in other words, we neglect the work due to her acceleration in the begining of the ride and deceleration at the ride's end.

With these assumptions the forward force on the bike must cancel backward forces due air drag and downhill component of the gravity force,

$$F_{\text{forward}} \;=\; mg \times \sin\theta \;+\; F_{\text{air drag}}\,.$$

Neglecting the air drag, we obtain

$$F_{\text{forward}} \;\approx\; mg \times \sin\theta = \;700\text{ N} \times 0.01 \;=\; 7\text{ N}. \tag{25}$$

Since this force has the same direction as the bike's displacement, its mechanical work is simply

$$W \;=\; F_{\text{forward}} \cdot L \;=\; 7\text{ N} \times 1000\text{ N} \;=\; 7000\text{ J}. \tag{26}$$

The forward force on the bike ultimately comes from the student pushing the bike's pedals, so assuming perfect efficiency of the bike as a machine converting the student's mechanical work into the work of the forward force, the student's work must be 7000 Joules.

Problem #**4**:

(a) Besides gravity, the forces acting on a roller coaster car are normal force N from the track, rolling friction f, and the air drag D. The normal force N is $\perp$ to the motion, so its mechanical work is zero. The rolling friction and the air drag perform negative work, but for this simplified problem we may neglect these forces and their work. Consequently, the net mechanical energy

$$E = K + U_{\text{grav}} = \tfrac{1}{2}mv^2 + mgy \tag{27}$$

of the car is conserved, $\Delta E = 0$.

At the starting point of the ride, the car has maximal elevation $y_0 = 78$ m and negligible initial speed $v_0 \approx 0$. Hence, the starting energy of the ride is

$$E = mgy_0 + 0, \tag{28}$$

and as the car rides down and up the coaster, its energy stays the same. Thus, if any particular point of the ride has elevation y, the speed of the car at that point follows from the mechanical energy conservation:

$$\tfrac{1}{2}mv^2 + mgy = E = 0 + mgy_0, \tag{29}$$

hence

$$\tfrac{1}{2}mv^2 = mgy_0 - mg_y = mg(y_0 - y) \tag{30}$$

and therefore

$$v = \sqrt{2g(y_0 - y)}\,. \tag{31}$$

Note that the car's mass cancels out form this formula!

In particular, at the top of the loop $y = 68$ m, hence the car goes through that point at speed

$$v = \sqrt{2 \times (9.8\ \text{m/s}^2) \times (78\ \text{m} - 68\ \text{m})} = \sqrt{19.6\ \text{m}^2/\text{s}^2} \approx 14\ \text{m/s} \approx 31\ \text{MPH}. \tag{32}$$

(b) To make sure an un-belted passenger does not fall out from an upside-down car, it must move with a downward acceleration greater than g. This will make the apparent weight of the passenger

$$\vec{W}_{\rm apparent} = m\vec{g} - m\vec{a} \tag{33}$$

point up rather than down — which would be towards his/her seat in the upside-down car and so keep him/her from falling out.

At the top of the loop, the loop's center is below the car, so its centripetal acceleration

$$a_c = \frac{v^2}{R} \tag{34}$$

is directed down. It is this downward acceleration that would keep passengers from falling out, *provided $a_c > g$.* Thus, a safe upside-down ride requires

$$a_c = \frac{v^2}{R} > g, \tag{35}$$

or equivalently

$$v > v_{\rm min} = \sqrt{Rg}. \tag{36}$$

In other words, at the top of the loop, the speed of a roller coaster car must be *faster than* $\sqrt{Rg}$.

For the loop in question, the minimal safe speed is

$$v_{\rm min} = \sqrt{(12\text{ m}) \times (9.8\text{ m/s}^2)} = \sqrt{118\text{ m}^2/\text{s}^2} \approx 11\text{ m/s} \approx 25\text{ MPH}. \tag{37}$$

Note: the speed $v = 14$ m/s we obtains in part (a) is faster than this minimal safe speed, so the ride should be OK.

Problem #**5**:

Since the cannon is loose, there are no large external forces on the cannon + ball system during the shooting. Consequently, in the brief time it takes the ball to leave the cannon, the impulse of the external forces is negligibly small, so the net momentum of the cannon and the ball

$$\vec{P}_{\text{net}} \;=\; m_{\text{ball}}\vec{v}_{\text{ball}} \;+\; m_{\text{cannon}}\vec{v}_{\text{cannon}} \tag{38}$$

is conserved,

$$\vec{P}_{\text{net}}(\text{just after the shooting}) = \vec{P}_{\text{net}}(\text{just before the shooting}). \tag{39}$$

Before the shooting, the cannon and the ball are both at rest, so their respective momenta are zero and hence $\vec{P}_{\text{net}} = 0$. Therefore, immediately after the shooting the net momentum remains zero, thus

$$m_{\text{ball}}\vec{v}_{\text{ball}} \;+\; m_{\text{cannon}}\vec{v}_{\text{cannon}} \;=\; 0. \tag{40}$$

Given both masses and the ball's velocity immediately after the shooting, we may solve this equation for the recoil velocity of the cannon:

$$\vec{v}_{\text{cannon}} \;=\; -\frac{m_{\text{ball}}}{m_{\text{cannon}}}\,\vec{v}_{\text{ball}}\,. \tag{41}$$

The minus sign here indicates the cannon recoiling in the opposite direction from the ball it shoots. In terms of speeds $v = |\vec{v}|$, the recoil speed of the cannon is

$$v_{\text{cannon}} \;=\; \frac{m_{\text{ball}}}{m_{\text{cannon}}} \times v_{\text{ball}}\,. \tag{42}$$

For the cannon in question

$$v_{\text{cannon}} \;=\; \frac{24\text{ pounds}}{1200\text{ pounds}} \times 500\text{ MPH} \;=\; 10\text{ MPH}, \tag{43}$$

fast enough to cause mayhem on a busy gundeck of a 1800-era battleship.