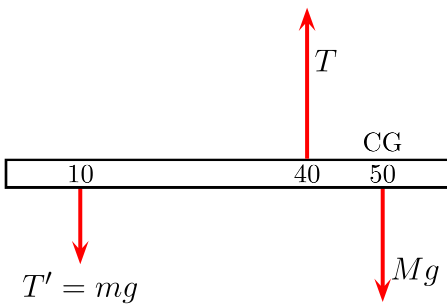
Problem #**1**:

There are three forces acting on the meter-stick: the tension T of the upper string, the tension $T' = mg$ (where $m = 50$ g) of the lower string, and the meter-stick's own weight Mg. Here is the force diagram:



Although the weight force Mg is distributed all over the meter-stick, for the purpose of calculating torques we treat it as acting at the center of gravity as shown on the above diagram. By symmetry, the center of mass is in the middle of the meter-stick, at the 50 cm mark.

The meter-stick is in equilibrium, so the net force and the net torque on it must be zero,

$$\sum F = 0, \qquad \sum \tau = 0. \tag{1}$$

In the torque condition, we may calculate the torques relative to any pivot point we like (as long as it's the same point for all the forces), so let's consider the net torque relative to the 40 cm mark where the upper string is attached. With this choice, the tension T of the upper string has zero lever arm, the tension $T' = mg$ of the lower string has lever arm 40 cm − 10 cm = 30 cm in the counterclockwise direction, and the meter-stick's own weight Mg has lever arm 50 cm − 40 cm = 10 cm in the clockwise direction. Consequently, the net torque is

$$\tau^{\text{net}} \equiv \tau(T) + \tau(mg) + \tau(Mg) = T \times 0 + mg \times 30\text{cm} - Mg \times 10\text{cm}. \tag{2}$$

In equilibrium this net torque vanishes, hence

$$mg \times\ 30\text{cm}\ -\ Mg \times\ 10\text{cm}\ =\ 0. \tag{3}$$

Solving this equation for the meter-stick's own mass M, we have

$$M\ =\ m \times \frac{30\text{ cm}}{10\text{ cm}}\ =\ m \times 3 =\ 50\text{ g} \times 3\ =\ 150\text{ g}. \tag{4}$$

<u>Problem #**2**:</u>

Out in space, there are no external torques on the two-spaceship system, so its net angular momentum is conserved,

$$L\ =\ I \times \omega\ =\ \text{const.} \tag{5}$$

Therefore, if the system's moment of inertia I changes for some reason, the angular velocity ω of its rotation must also change such that the angular momentum remains constant:

$$I \to I', \quad \omega \to \omega', \quad \text{but} \quad I' \times \omega'\ =\ L'\ =\ L\ =\ I \times \omega. \tag{6}$$

The moment of inertia of the two-spaceship system depends on the cable's length ℓ. Each spaceship is at distance $r = \frac{1}{2}\ell$ from the system center of mass — around which it rotates — so it contributes $Mr^2 = M(\ell/s)^2$ to the system's moment of inertia. The net moment of inertia is therefore

$$I\ =\ M(\ell/2)^2\ +\ M(\ell/2)^2\ =\ \tfrac{1}{2}M\ell^2. \tag{7}$$

When the astronauts pull on the cable and shorten the distance between the two spaceships from $\ell = 240$ m to $\ell' = 120$ m, they reduce the moment of inertia from $I = \frac{1}{2}M\ell^2$ to $I' = \frac{1}{2}M\ell'^2$. Consequently, the system's angular velocity changes to maintain constant angular momentum; solving eq. (6) for the ω', we have

$$\omega' \;=\; \omega \times \frac{I}{I'} \tag{8}$$

where

$$\frac{I}{I'} \;=\; \frac{\frac{1}{2}M\ell^2}{\frac{1}{2}M\ell'^2} \;=\; \frac{\ell^2}{\ell'^2} \;=\; \left(\frac{\ell}{\ell'}\right)^2 \;=\; \left(\frac{240\text{ m}}{120\text{ m}} = 2.0\right)^2 \;=\; 4.0, \tag{9}$$

hence

$$\omega' \;=\; 4.0 \times \omega, \tag{10}$$

the system now rotates 4 times faster than before. Numerically,

$$\omega' \;=\; 8.7 \cdot 10^{-3}\text{ rad/s} \times 4.0 \;=\; 35 \cdot 10^{-3}\text{ rad/s}, \tag{11}$$

or 1 revolution every 3 minutes.

Problem **#3**:

By the Archimedes's Law, the buoyant force on the floating plastic globe equals the weight of the water it displaces,

$$F_b \;=\; V_{\text{disp}} \times \rho_w \times g. \tag{12}$$

Note that the V_{disp} in this formula is not the volume of the whole globe but only of its submerged part that displaces the water. A globe floating equator-deep has the bottom hemisphere submerged while the top hemisphere is above the waterline, so the volume of the displaced water is one half of the whole ball's volume $V = 4400\text{ cm}^3$,

$$V_{\text{subm}} \;=\; \frac{V}{2} \;=\; 2200\text{ cm}^3. \tag{13}$$

Hence, the buoyant force is

$$F_b \;=\; \frac{V}{2} \times \rho_w \times g.$$

Since the globe floats in equilibrium, this buoyant force must cancel the globe's own weight,

$$F_y^{\mathrm{bet}} \;=\; F_b \;-\; mg \;=\; 0. \tag{14}$$

Consequently

$$mg \;=\; F_b \;=\; \frac{V}{2}\times\rho_w\times g, \tag{15}$$

so the globe's mass must be

$$m \;=\; \frac{V}{2}\times\rho_w \;=\; 2200\ \mathrm{cm}^3\times 1.0000\ \mathrm{g/cm}^3 \;=\; 2200\ \mathrm{g}. \tag{16}$$

Finally, the density of the plastic making up this globe is the ratio of the globe's mass to the volume filled with plastic. The globe in question is solid, so its whole volume $V = 4400\ \mathrm{cm}^3$ is filled with plastic, hence plastic's density is

$$\rho \;=\; \frac{m}{V} \;=\; \frac{2200\ \mathrm{g}}{4400\ \mathrm{cm}^3} \;=\; 0.50\ \mathrm{g/cm}^3. \tag{17}$$

Problem #**4**:

(a) The speed of water jetting out from the bottle obtains from the Bernoulli equation

$$P_1 \;+\; \rho g y_1 \;+\; \tfrac{1}{2}\rho v_1^2 \;=\; P_2 \;+\; \rho g y_2 \;+\; \tfrac{1}{2}\rho v_2^2 \tag{18}$$

for any two points 1 and 2 along the water flow. Let point 1 be the surface of water in the bottle while 2 is the opening at the bottle's bottom from which the water jets out. Then v_2 is the speed of the water jet we need to find out in part (a).

To solve the Bernoulli equation (18) for the speed v_2, let's reorganize the equation as

$$\frac{\rho}{2}\times\left(v_2^2 \;-\; v_1^2\right) \;=\; \left(P_1 - P_2\right) \;+\; \rho g\times(y_1 - y_2). \tag{19}$$

On the left hand side we may relate the speeds v_1 and v_2 by the continuity equation

$$v_1\times A_1 \;=\; \mathcal{F} \;=\; v_2\times A_2 \tag{20}$$

where A_2 is the cross-sectional area of the bottle's opening while A_1 is the cross-sectional area of the bottle itself. Since the opening is much narrow than the bottle itself, $A_2 \ll A_1$, the continuity equation tells us that the water flows through the bottle itself much slower than through the opening,

$$v_1 \;=\; \frac{A_2}{A_1}\times v_2 \;\ll\; v_2\,. \tag{21}$$

Consequently, on the left hand side of eq. (19), we may neglect the v_1^2 term compared to the v_2^2 term, thus $v^2 - v_1^2 \approx v_2^2$, and hence

$$\frac{\rho}{2}\times v_2^2 \;\approx\; \bigl(P_1 - P_2\bigr) \;+\; \rho g\times(y_1 - y_2). \tag{22}$$

As to the right hand side of this equation, the problem tells us to neglect the height $y_1 - y_2$ of the water inside the bottle, which means neglecting the $\rho g(y_1 - y_2)$ term compared to the $(P_1 - P_2)$, hence

$$\frac{\rho}{2}\times v_2^2 \;\approx\; P_1 \;-\; P_2\,. \tag{23}$$

On the right hand side of this simplified equation, P_2 is the water pressure in the jet emerging from the bottle; since the jet is open to the air, P_2 = air pressure outside the bottle. On the other hand, P_1 is the water pressure inside the bottle, which is equal to the pressure of compressed air inside the bottle. Consequently,

$$P_1 \;-\; P_2 \;=\; P^{\text{air}}_{\text{inside}} \;-\; P^{\text{air}}_{\text{outside}} \;=\; P^{\text{gauge}}, \tag{24}$$

the *gauge pressure* of the air in the bottle; the problem tells us this gauge pressure is 690 kilo-Pascals. Thus, eq. (23) becomes

$$\frac{\rho}{2}\times v_2^2 \;=\; P^{\text{gauge}} \;=\; 690\ \text{kPa} \tag{25}$$

where ρ is the density of water, $\rho = 1000.0\ \text{kg/m}^3$. Solving eq. (25), we obtain

$$v_2^2 \;=\; \frac{2P^{\text{gauge}}}{\rho} \;=\; \frac{2\times 690,000\ \text{Pa}}{1,000.0\ \text{kg/m}^3} \;=\; 1380\ \text{J/kg} \;=\; 1380\ \text{m}^2/\text{s}^2 \tag{26}$$

and hence

$$v_2 \;=\; \sqrt{1380\ \text{m}^2/\text{s}^2} \;\approx\; 37\ \text{m/s}, \tag{27}$$

about 83 miles per hour.

(b) Given the speed of the water jet we have obtained in part (b) and the cross-sectional area $A_2 = 3.8\ \mathrm{cm}^2$ of the bottle opening, we can find the volume flow rate

$$\mathcal{F} \;=\; v_2 \times A_2 \;=\; (37\ \mathrm{m/s}) \times (3.8 \cdot 10^{-4}\ \mathrm{m}^2) \;\approx\; 14 \cdot 10^{-3}\ \mathrm{m}^3/\mathrm{s} \;=\; 14\ \mathrm{L/s}, \tag{28}$$

almost 4 gallons per second.

The mass flow rate of water follows by multiplying the volume flow rate by the water density,

$$\frac{M}{t} \;=\; \mathcal{F} \times \rho \;=\; (14 \cdot 10^{-3}\ \mathrm{m}^3/\mathrm{s}) \times (1000.0\ \mathrm{kg/m}^3) \;=\; 14\ \mathrm{kg/s}. \tag{29}$$

(c) The *rocket equation* gives the thrust force of a rocket as a product of the exhaust speed of the jet it produces times the mass flow rate of the jet,

$$F_{\mathrm{thrust}} \;=\; v_{\mathrm{exhaust}} \times \frac{M}{t}\,. \tag{30}$$

For the water rocket in question, the exhaust speed is the speed of the water jet $v_2 = 37$ m/s we found in part (a), while the mass flow rate was found in part (b) to be 14 kg/s. Therefore, the thrust of this rocket is

$$F_{\mathrm{thrust}} \;=\; (37\ \mathrm{m/s}) \times (14\ \mathrm{kg/s}) \;=\; 520\ \mathrm{kg} \cdot \mathrm{m/s}^2 \;=\; 520\ \mathrm{N}, \tag{31}$$

about 120 pounds.

PS: In case you forgot the rocket equation, let me re-derive it from the momentum-impulse theorem and the law of momentum conservation. During a short time interval Δt, the rocket ejects mass

$$\Delta M \;=\; \frac{M}{t} \times \Delta t \tag{32}$$

of something, usually burned-up rocket fuel, but in our case just water. The velocity of the ejected mass is $-v_e$ relative to the rocket (where the $-$ sign indicates the backward direction), so its momentum changes by

$$\Delta p[\text{ejected}] \;=\; -v_e \times \Delta m. \tag{33}$$

But the net momentum of the rocket+ejected material system is conserved, so the rocket's momentum changes by the opposite amount,

$$\Delta p[\text{rocket}] \;=\; +v_e \times \Delta m \;=\; +v_e \times \frac{M}{t} \times \Delta t. \tag{34}$$

As far as the rocket is concerned, this change of momentum is the impulse of the thrust force of the rocket engine, thus

$$\delta p[\text{rocket}] \;=\; F_{\text{thrust}} \times \Delta t. \tag{35}$$

Comparing the last two equations, we immediately obtain

$$F_{\text{thrust}} \times \Delta t \;=\; \Delta p[\text{rocket}] \;=\; +v_e \times \frac{M}{t} \times \Delta t \tag{36}$$

and hence the rocket equation

$$F_{\text{thrust}} \;=\; v_e \times \frac{M}{t}\,. \tag{30}$$

Problem #**5**:

(a) The rules for converting temperatures from the Fahrenheit scale to the Celsius (centigrade) scale and hence to the absolute Kelvin scale (counting from the absolute zero) are

$$\begin{aligned} T[\text{in }^\circ\text{C}] &= \frac{5}{9} \times \bigl(T[\text{in }^\circ\text{F}] - 32\bigr), \\ T[\text{in K}] &= T[\text{in }^\circ\text{C}] + 273.15. \end{aligned} \tag{37}$$

Thus, the ground-level temperature converts to

$$T_1 \;=\; 95^\circ\text{ F} \;=\; 35^\circ\text{ C} \;=\; 308.15\text{ K}, \tag{38}$$

while the temperature at the 20,000 ft altitude converts to

$$T_2 \;=\; 14^\circ\text{ F} \;=\; -10^\circ\text{ C} \;=\; 263.15\text{ K}. \tag{39}$$

(b) The combined gas law tells us that for a fixed amount of gas, its volume P, the absolute pressure P, and the absolute temperature T (in Kelvins) are related to each other so that

$$\frac{P \times V}{T} = \text{const} \tag{40}$$

Therefore, knowing the initial volume, pressure, and temperature of the helium gas in the balloon in question, we may relate its volume, pressure, and temperature at some later time as

$$\frac{P_2 \times V_2}{T_2} = \frac{P_1 \times V_1}{T_1}. \tag{41}$$

In particular, we may find the volume of the rising balloon from the pressure and temperature at its altitude by solving eq. (41) for the V_2:

$$\begin{aligned} \frac{P_2 \times V_2}{T_2} &= \frac{P_1 \times V_1}{T_1} \\ &\Downarrow \\ V_2 \times \frac{P_2}{T_2} &= V_1 \times \frac{P_1}{T_1} \\ &\Downarrow \\ V_2 &= \left(V_1 \times \frac{P_1}{T_1}\right) \times \frac{T_2}{P_2} = V_1 \times \frac{P_1}{P_2} \times \frac{T_2}{T_1}. \end{aligned} \tag{42}$$

Note that the helium pressures P_1 and P_2 in this formula must be absolute pressures rather than gauge pressures. Likewise, the temperatures T_1 and T_2 must be absolute temperature, *i.e.*, counted from the absolute zero. In other words, we should convert the temperatures from the relative Fahrenheit scale to the absolute Kelvin scale, just as we did in part (a). Thus, in eq. (42), $P_1 = 101.3$ kPa, $P_2 = 46.4$ kPa, $T_1 = 308.15$ K, $T_2 = 263.15$ K, therefore

$$V_2 = V_1 \times \frac{101.3 \text{ kPa}}{46.6 \text{ kPa}} \times \frac{263.15\ K}{308.15\ K} = V_1 \times 1.856 \tag{43}$$

— the balloon's volume at the 20,000 ft altitude became 85.6% larger than at the ground level. In absolute terms, $V_1 = 125\ \text{m}^3$ while

$$V_2 = 1.856 \times 125\ \text{m}^3 = 232\ \text{m}^3. \tag{44}$$