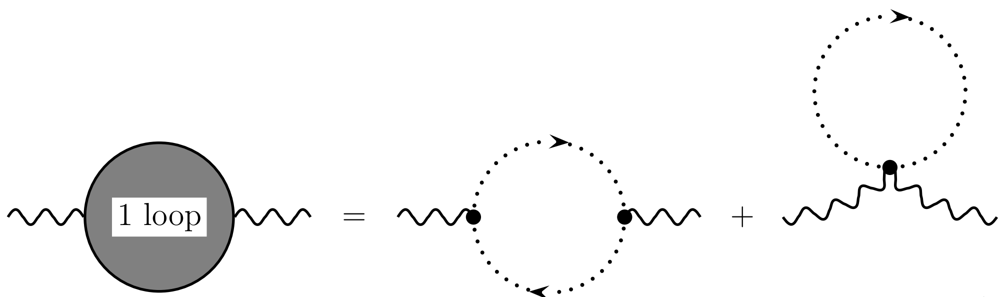
1. First, finish the textbook problem **10.2** — calculate to one-loop order the infinite parts of all the counterterms of the pseudoscalar Yukawa theory.

   Hint: the infinite part of the four-scalar amplitude $iV(k_1, \ldots, k_4)$ does not depend on the scalar's momenta, so you may calculate it for any particular $k_1, \ldots, k_4$ you like, on-shell of off-shell. I suggest you take $k_1 = k_2 = k_3 = k_4 = 0$, so in any one-loop diagram all the propagators in the loop have the same momentum q — which makes evaluating such a diagram much simpler.

   Likewise, the infinite part of the one-scalar-two-fermions amplitude $\Gamma^5(p', p)$ does not depend on the momenta p, p', or $k = p' - p$, so you may calculate it for any p and p' you like, on-shell or off shell. Again, letting $p = p' = 0$ makes for a much simpler calculation of the one-loop diagram(s).

2. And now consider the electric charge renormalization in the scalar QED — the theory of a EM field A^μ interacting with a charged scalar field Φ. At the one-loop level, there are two Feynman diagrams contributing to the 1PI two-photon amplitude, namely

$i\Sigma^{\mu\nu}_{1\,\text{loop}} =$ 

(1)

   (a) Evaluate the two diagrams using dimensional regularization and verify that the net amplitude has form

$$\Sigma^{\mu\nu}_{1\,\text{loop}}(k) = \left(k^2 g^{\mu\nu} - k^\mu k^\nu\right) \times \Pi_{1\,\text{loop}}(k^2) \tag{2}$$

(b) Calculate the $\Pi(k^2)$ due to the one-loop diagrams (1), determine the δ_3 counterterm (at the $O(e^2)$ level), and write down the *net* $\Pi(k^2)$ as a function of k^2.

(c) Finally, consider the effective coupling $\alpha_{\text{eff}}(k^2)$ of the scalar QED at high momenta. Show that at the one-loop level,

$$\frac{1}{\alpha_{\text{eff}}(k^2)} \approx \frac{1}{\alpha(0)} - \frac{1}{12\pi}\left(\log\frac{-k^2}{m^2} - \frac{8}{3}\right). \tag{3}$$