

1. In class, I have calculated the (infinite parts of the) $\delta_2^{(q)}$ and $\delta_1^{(q)}$ counterterms for the quarks. Your task is to calculate the analogous $\delta_2^{(\text{gh})}$ and $\delta_1^{(\text{gh})}$ counterterms for the *ghosts fields*.
 - (a) Draw one-loop diagrams whose divergences are cancelled by the $\delta_2^{(\text{gh})}$ and $\delta_1^{(\text{gh})}$, and calculate the group factors for each diagrams.
 - (b) Calculate the momentum integrals for the diagrams. Focus on the UV divergences and ignore the finite parts of the integrals.
 - (c) Assemble your results and show that the *difference* $\delta_1^{(\text{gh})} - \delta_2^{(\text{gh})}$ for the ghosts is exactly the same as the $\delta_1^{(q)} - \delta_2^{(q)}$ difference for the quarks.
2. Now consider the three gauge couplings of the $SU(3) \times SU(2) \times U(1)$ Standard Model and their one-loop beta-functions

$$\beta_1^{1\text{loop}} = \frac{b_1 g_1^3}{16\pi^2}, \quad \beta_2^{1\text{loop}} = \frac{b_2 g_2^3}{16\pi^2}, \quad \beta_3^{1\text{loop}} = \frac{b_3 g_3^3}{16\pi^2}. \quad (1)$$

In this exercise, you do not need to calculate these beta-function from scratch by evaluating the UV divergences of a bunch of loop diagrams. Instead, use eqs. (84–85) from the last page of [my notes on QCD beta-function](#).

- (a) Calculate the b_1, b_2, b_3 coefficients it for the minimal version of the Standard Model: the $SU(3) \times SU(2) \times U(1)$ gauge fields, one Higgs doublet, three families of quarks and leptons, and nothing else.
 - ★ FYI, each family comprises 8 left-handed Weyl fields in the $(\mathbf{3}, \mathbf{2}, y = +\frac{1}{6})$ and $(\mathbf{1}, \mathbf{2}, y = -\frac{1}{2})$ multiplets of the gauge symmetry and 7 right-handed Weyl fermions in the $(\mathbf{3}, \mathbf{1}, y = +\frac{2}{3})$, $(\mathbf{3}, \mathbf{1}, y = -\frac{1}{3})$, and $(\mathbf{1}, \mathbf{1}, y = -1)$ multiplets.
- (b) Re-calculate the b_1, b_2, b_3 for the MSSM — the Minimal Supersymmetric Standard Model. FYI, here is complete list of the MSSM fields:
 - The $SU(3) \times SU(2) \times U(1)$ gauge fields, same as the non-SUSY SM.
 - For each vector field there is a Majorana fermion (gaugino) with similar $SU(3) \times SU(2) \times U(1)$ quantum numbers. Altogether, there is an adjoint multiplet of gauginos for each factor of the gauge symmetry.

- 3 families of quarks and leptons, same as the non-SUSY SM.
 - For each Weyl fermion — left-handed or right-handed — in these three families, the MSSM also have a complex scalar field (squark or slepton) with similar $SU(3) \times SU(2) \times U(1)$ quantum numbers. Altogether, this makes 45 complex scalar fields in similar multiplets to the quarks and leptons.
 - The Higgs sector of the MSSM comprises *two* $SU(2)$ doublets of complex scalars accompanied by one $SU(2)$ doublet of Dirac fermions (the higgsinos); all these doublets have $y = \frac{1}{2}$.
- There are all kinds of Yukawa and ϕ^4 interactions between the MSSM fields, but you do not need them for the one-loop calculation of the gauge couplings' beta-functions.

In Grand Unified Theories

$$\alpha_3 = \alpha_2 = \frac{5}{3}\alpha_1 = \alpha_{\text{GUT}} \quad \text{at the GUT scale.} \quad (2)$$

At lower energy scales $E \ll M_{\text{GUT}}$ the SM couplings are given (to the leading one-loop order) by the Georgi–Quinn–Weinberg equations

$$\begin{aligned} \frac{1}{\alpha_3(E)} &= \frac{1}{\alpha_{\text{GUT}}} + b_3 \times \frac{1}{2\pi} \log \frac{M_{\text{GUT}}}{E}, \\ \frac{1}{\alpha_2(E)} &= \frac{1}{\alpha_{\text{GUT}}} + b_2 \times \frac{1}{2\pi} \log \frac{M_{\text{GUT}}}{E}, \\ \frac{1}{\alpha_1(E)} &= \frac{5/3}{\alpha_{\text{GUT}}} + b_1 \times \frac{1}{2\pi} \log \frac{M_{\text{GUT}}}{E}. \end{aligned} \quad (3)$$

(c) Derive these equations from eqs. (1).

(d) Experimentally, at $E = M_Z \approx 91$ GeV

$$\frac{1}{\alpha_3(M_Z)} \approx 8.5, \quad \frac{1}{\alpha_2(M_Z)} \approx 29.5, \quad \frac{1}{\alpha_1(M_Z)} \approx 98.5. \quad (4)$$

Check that these couplings are consistent with eqs. (3) for the MSSM but not for the non-SUSY minimal Standard Model. For the MSSM, calculate the Grand Unification scale M_{GUT} and the unified gauge coupling α_{GUT} .

Although the top quark, the Higgs, and all the additional particles of the MSSM are heavier than Z , for this exercise you should ignore the thresholds due to these masses. Instead, use the b_3, b_2, b_1 coefficients of the massless theory — the minimal SM or the MSSM — for all energies between the M_Z and the M_{GUT} .

3. Next, a reading assignment: §16.7 of *Peskin & Schroeder* about the “magnetic anti-screening” explanation of the asymptotic freedom in QCD.
4. Finally, another reading assignment: §19.3 of *Peskin & Schroeder* about the chiral symmetry of QCD and the pions.

Chapter 19 of *Weinberg* has a deeper discussion of pions (and Goldstone bosons in general); you are advised to read it, but not necessarily this week.