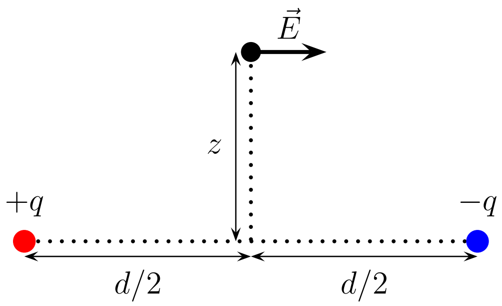
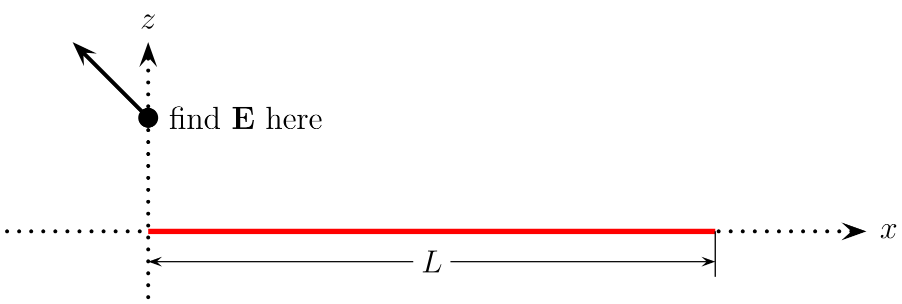
Problem 2.2: Find the electric field **E** (both the magnitude and the direction) at distance z above the midpoint between two equal and opposite point charges $\pm q$, at distance d apart from each other.

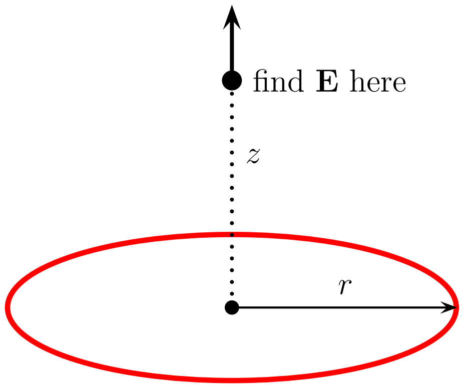


Problem 2.3: Find the electric field **E** directly above one end of a straight line segment of length L that carries a uniform line charge λ.

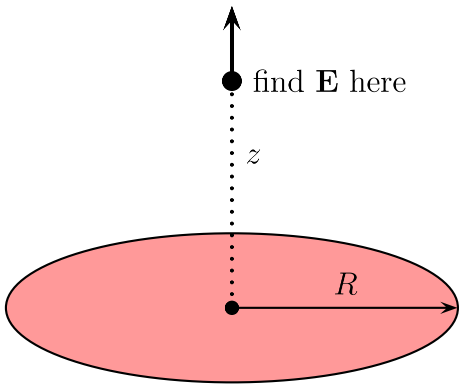


Check that your formula is consistent with what you would expect for $z \gg L$.

Problem 2.5: Find the electric field at height z directly above the center of a circular loop of radius r carrying uniform line λ.



Problem 2.6: Find the electric field at height z directly above the center of a flat circular disk of radius R that carries a uniform surface charge σ.



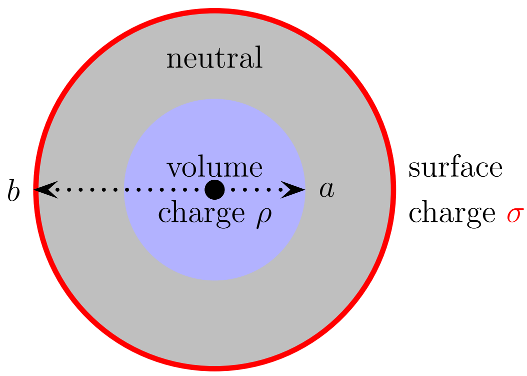
What does your formula give in the limit of $R \to \infty$? Also, check the opposite limit of $z \gg R$.

Problem 2.15: A thick spherical shell of inner radius a and outer radius b carries a non-uniform (but spherically symmetric) charge density

$$\rho(r) = \frac{k}{r^2} \quad [\text{for } a \leq r \leq b \text{ only}].$$

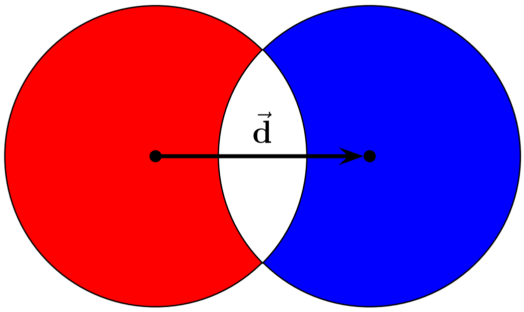
Find the electric field in the three regions: (i) $r < a$, (ii) $a < r < b$, (iii) $r > b$. Plot $|\mathbf{E}|$ as a function of the radius r for the case of $b = 2a$.

Problem 2.16: Conside a *long* coaxial cable comprised of the inner cylinder of radius a and the outer cylindric shell from a to b. The inner cylinder carries a uniform *volume* charge density ρ. The outer shell has no volume charges but on its outer surface (at $r = b$) it has a uniform *surface* charge density σ. The sign of σ is opposite to ρ and its magnitude is such that the whole cable has zero net charge. Here is what the cable's cross-section looks like:



Find the electric in each of the three regions: (i) $r < a$, inside the inner cylinder; (ii) $a < r < b$, inside the outer shell; (iii) $r > b$, outside the cable. Plot $|\mathbf{E}|$ as a function of the radius r.

Problem 2.18: Two spheres of the same radius R and containing equal and opposite volume charge densities — respectively, $+\rho$ and $-\rho$ — are placed so that they partially overlap. The overlapping region is rendered neutral.



Show that the electric field in the overlapping region is uniform and calculate its value as a function of the vector $\mathbf{d}$ from the positive center to the negative center.
Hint: first, use the Gauss Law to calculate the electric field inside a uniformly charged solid sphere, *cf.* problem 2.12.

Problem 1.6: Prove that

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \ + \ \mathbf{B} \times (\mathbf{C} \times \mathbf{A}) \ + \ \mathbf{C} \times (\mathbf{A} \times \mathbf{B}) \ = \ \mathbf{0}.$$

Also, under what conditions $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$?

Problem 1.11: Find the gradients of the following functions:

$$\begin{aligned}
(a) \quad f(x, y, z) &= x^2 + y^3 + z^4. \\
(b) \quad f(x, y, z) &= x^2 y^3 z^4. \\
(c) \quad f(x, y, z) &= e^x \sin(y) \ln(z).
\end{aligned}$$

Problem 1.13: Let $\vec{\mathcal{R}} = \mathbf{r} - \mathbf{R}_0$ be the radius vector from a fixed point $\mathbf{R}_0 = (X_0, Y_0, Z_0)$ to the point $\mathbf{r} = (x, y, z)$ we follow, and let $\mathcal{R} = |\vec{\mathcal{R}}|$ be its length, *i.e.* the distance between the points $\mathbf{R}_0$ and $\mathbf{r}$. Consider the gradients of powers of this distance $\mathcal{R}$ with respect to $\mathbf{r}$ (while the $\mathbf{R}_0$ is held fixed). Show that

$$(a) \quad \nabla(\mathcal{R}^2) = 2\vec{\mathcal{R}};$$

$$(b) \quad \nabla(1/\mathcal{R}) = -\widehat{\mathcal{R}}/\mathcal{R}^2;$$

(c) what is the general formula for the $\nabla\big(\mathcal{R}^n\big)$?