1. Consider EM radiation generated by a charged particle moving in a circle of radius R at frequency ω. Assume $R\omega \ll c$, so you may use the electric dipole approximation.

   (a) What is the angular distribution of the power generated by the rotating particle?

   Now consider the classical Rutherford model of a hydrogen-like atom or ion: an electron moving in a circular orbit around a nucleus of charge $+Ze$.

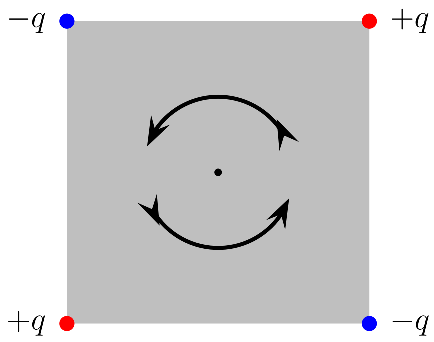
   (b) Find the net power of EM radiation generated by the electron in a circular orbit of radius R.

   (c) As the electron loses some of its energy to the radiation, its orbit becomes smaller. Show that the electron takes only a finite time to 'fall down' all the way to the nucleus — even in the limit of nuclear size $\to 0$ — and calculate this time in terms of the initial orbital radius.

   (d) Suppose the classical atom had the same initial energy as a quantum atom at energy level $n = 2$; how long would it live before the electron would crash into the nucleus?

   Look up in the literature or on the Internet the quantum transition rate between the hydrogen states 2P and 1S and compare to classical lifetime.

2. Four charges $\pm q$ sit at corners of a square of size $a \times a$, which rotates with frequency ω around the $\perp$ axis through the square's center.



   (a) Find the electric quadrupole moment tensor of this system. With what frequency does it rotate?

(b) Find the angular distribution of the EM power radiated by the rotating quadrupole.

(c) Find the net EM power radiated by the rotating quadrupole.

3. Consider an electric dipole with a time-dependent dipole moment $\mathbf{d}(t)$. In a pure dipole approximation, the charge density and the current density of this system become

$$\rho(\mathbf{y},t) \;=\; -\big(\mathbf{d}(t)\cdot\nabla_y\big)\delta^{(3)}(\mathbf{y}), \quad \mathbf{J}(\mathbf{y},t) \;=\; \frac{d\,\mathbf{d}}{dt}\,\delta^{(3)}(\mathbf{y}) \tag{1}$$

(a) Use the retarded Green's function to show that in the Landau gauge

$$\Phi(\mathbf{x},t) \;=\; \frac{1}{4\pi\epsilon_0}\left[\frac{\mathbf{n}}{r^2}\cdot\mathbf{d} + \frac{\mathbf{n}}{rc}\cdot\frac{d\,\mathbf{d}}{dt}\right]_{\text{ret}}, \qquad \mathbf{A}(\mathbf{x},t) \;=\; \frac{\mu_0}{4\pi r}\left[\frac{d\,\mathbf{d}}{dt}\right]_{\text{ret}}, \tag{2}$$

where $\mathbf{d}$ and its time derivative should be evaluated at the retarded time $t_{\text{ret}} = t - r/c$.

(b) Calculate the electric and the magnetic field for a general $\mathbf{d}(t)$.

(c) Now assume the harmonic time dependence $\mathbf{d}(t) = \mathbf{d}\exp(-i\omega t)$. Show that in this case

$$\begin{aligned} c\mathbf{B}(\mathbf{x},t) \;&=\; \frac{k^2}{4\pi\epsilon_0}\,\frac{e^{ikr-i\omega t}}{r}\left(1+\frac{i}{kr}\right)(\mathbf{n}\times\mathbf{d}), \\ \mathbf{E}(\mathbf{x},t) \;&=\; \frac{k^2}{4\pi\epsilon_0}\,\frac{e^{ikr-i\omega t}}{r}\left[\frac{i}{kr}\left(1+\frac{i}{kr}\right)\big(\mathbf{d} - 3(\mathbf{n}\cdot\mathbf{d})\mathbf{n}\big) - \mathbf{n}\times(\mathbf{n}\times\mathbf{d})\right]. \end{aligned} \tag{3}$$

(d) Explain the long-distance and the short-distance limits of the EM fields (3).