QCD Feynman Rules

Quarks are Dirac fermion fields Ψ_{if} which come in 3 colors $i = 1,2,3$ and 6 flavors $f = u,d,s,c,b,t$. The $SU(3)$ symmetry of the 3 colors is an exact local symmetry, so it comes with $3^2 - 1 = 8$ vector fields $A^a_\mu(x)$ — the gluons. The semi-classical Lagrangian of the theory is quite simple, especially in the color-matrix notations:

$$\mathcal{L} \;=\; -\tfrac{1}{2}\,\mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) \;+\; \sum_f \overline{\Psi}_f\bigl(i\gamma^\mu D_\mu \;-\; m_f\bigr)\Psi_f \tag{1}$$

where $D_\mu\Psi = \partial_\mu\Psi + igA_\mu\Psi$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$. In the explicit-color notations, the formulae are slightly more complicated:

$$\mathcal{L} \;=\; -\tfrac{1}{4}\,F^a_{\mu\nu}F^{a\mu\nu} \;+\; \sum\nolimits_f \overline{\Psi}_{f,i}\bigl(i\gamma^\mu D_\mu \;-\; m_f\bigr)\Psi^i_f\,, \tag{2}$$

$$D_\mu\Psi^i_f \;=\; \partial_\mu\Psi^i_f \;+\; \frac{ig}{2}\,A^a_\mu\bigl(\lambda^a\bigr)^i{}_j\Psi^j_f\,, \tag{3}$$

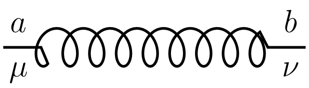
$$F^a_{\mu\nu} \;=\; \partial_\mu A^a_\nu \;-\; \partial_\nu A^a_\mu \;-\; gf^{abc}A^b_\mu A^c_\nu\,. \tag{4}$$

To set up the perturbation theory, we expand this Lagrangian in powers of the gauge coupling g:

$$\begin{aligned}\mathcal{L} \;=\; &-\tfrac{1}{4}\bigl(\partial_\mu A^a_\nu \;-\; \partial_\nu A^a_\mu\bigr)^2 + \sum_f \overline{\Psi}_{f,i}(i\gamma^\mu\partial_\mu - m_f)\Psi^i_f \\ &+\; gf^{abc}(\partial_\mu A^a_\nu)A^b_\mu A_\nu c \;-\; \frac{g^2}{4}\,f^{abc}f^{ade}\,A^b_\mu A^c_\nu A^d_\mu A^e_\nu \;+\; \frac{ig}{2}\,A^a_\mu \times \sum\nolimits_f \overline{\Psi}_{f,i}\gamma^\mu(\lambda^a)^i{}_j\Psi^j_f\,.\end{aligned} \tag{5}$$

The first line of this expansion describes the free gluon and quark fields, while the second line describes their interactions. In Feynman rules, the propagators follow from the first line of eq. (5), while the 3–gluon, 4–gluon, and gluon-quark-antiquark vertices follow from the second line. Altogether, we have:

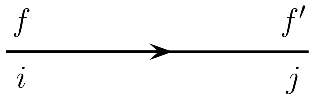
- The gluon propagator



$$= \frac{-i\delta^{ab}}{k^2 + i0}\left(g^{\mu\nu} \;+\; (\xi - 1)\frac{k^\mu k^\nu}{k^2 + i0}\right) \tag{6}$$

  where ξ is the gauge-fixing parameter. For $\xi = 0$ we have the Landau gauge while for $\xi = 1$ — the Feynman gauge.

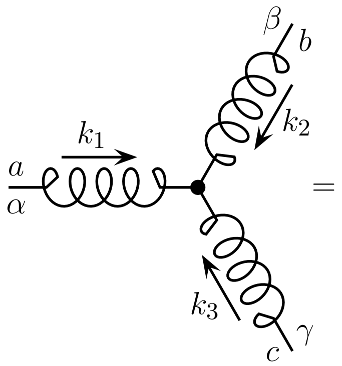
- The quark propagator



$$= \frac{i\delta^i_j \delta_{ff'}}{\not{p} - m_f + i0}\,. \tag{7}$$

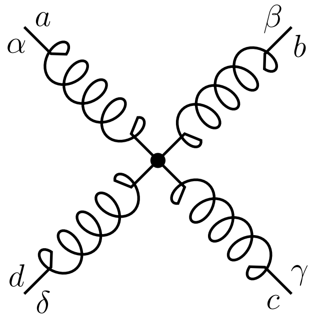
Note: the colors and the flavors must be the same at both ends of the propagator.

- The three-gluon vertex



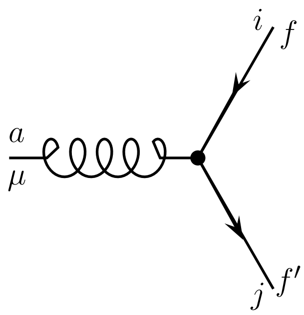
$$= \; -gf^{abc}\left[g^{\alpha\beta}(k_1-k_2)^\gamma \;+\; g^{\beta\gamma}(k_2-k_3)^\alpha \;+\; g^{\gamma\alpha}(k_3-k_1)^\beta\right]. \tag{8}$$

- The four-gluon vertex



$$= \; -ig^2 \begin{bmatrix} f^{abe}f^{cde}(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}) \\ +\, f^{ace}f^{bde}(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\delta}g^{\gamma\beta}) \\ +\, f^{ade}f^{bce}(g^{\alpha\beta}g^{\delta\gamma} - g^{\alpha\gamma}g^{\delta\beta}) \end{bmatrix}. \tag{9}$$

- The quark-antiquark-gluon vertex



$$= \; -ig\gamma^\mu \times \delta_{ff'} \times \left(\frac{\lambda^a}{2}\right)^j_i\,. \tag{10}$$

Note: the quark lines connected to the vertex must have the same flavors $f' = f$ but they may have different colors $j \neq i$.

⋆ The external line factors and the sign rules of QCD are exactly the same as in QED.

Actually, the above Feynman rules are OK at the tree level but insufficient for the loop calculations. The problem stems from the gauge fixing the gluon fields in order to quantize them canonically and get the gluon propagator. In QED, simple linear constraints like $\nabla \cdot \mathbf{A} = 0$ or $\partial_\mu A^\mu = 0$ on the abelian gauge field were harmless, but for the non-abelian gluon fields of QCD such linear constraints create all kinds of problems at the loop level. In the path integral formalism, such constraints screw up the measure of the path integral, but we can un-screw it by introducing additional un-physical fields called the *ghosts*. In Feynman rules, there are ghost propagators and ghost-ghost-gluon vertices, but no external ghost lines — the ghosts may run in loops but never as external particles. We shall deal with the "ghostly" aspects of the QCD Feynman rules in the QFT (II) class in Spring 2017.