

1. First, a reading assignment: [my notes on Ward–Takahashi identities](#). Please go *carefully* through the algebra, and make sure you understand the diagrammatic proof of the identities.

I strongly suggest you work through the first $17\frac{1}{2}$ pages on my notes by Monday 2/25.

2. Next, finish the textbook problem **10.2** — calculate to one-loop order the infinite parts of all the counterterms of the pseudoscalar Yukawa theory.

Hint: the infinite part of the four-scalar amplitude $iV(k_1, \dots, k_4)$ does not depend on the scalar's momenta, so you may calculate it for any particular k_1, \dots, k_4 you like, on-shell or off-shell. I suggest you take $k_1 = k_2 = k_3 = k_4 = 0$, so in any one-loop diagram all the propagators in the loop have the same momentum q — which makes evaluating such a diagram much simpler.

Likewise, the infinite part of the one-scalar-two-fermions amplitude $\Gamma^5(p', p)$ does not depend on the momenta p, p' , or $k = p' - p$, so you may calculate it for any p and p' you like, on-shell or off shell. Again, letting $p = p' = 0$ makes for a much simpler calculation of the one-loop diagram(s).

3. And now consider the electric charge renormalization in the scalar QED — the theory of a EM field A^μ interacting with a charged scalar field Φ . At the one-loop level, there are two Feynman diagrams contributing to the 1PI two-photon amplitude, namely

$$i\Sigma_{1\text{loop}}^{\mu\nu} = \text{[Diagram: wavy line to a grey circle labeled '1 loop', wavy line]} = \text{[Diagram: wavy line to a loop of dashed lines with arrows, wavy line]} + \text{[Diagram: wavy line to a loop of dashed lines with arrows, wavy line]} \quad (1)$$

- (a) Evaluate the two diagrams using dimensional regularization and verify that the net

amplitude has form

$$\Sigma_{1\text{loop}}^{\mu\nu}(k) = (k^2 g^{\mu\nu} - k^\mu k^\nu) \times \Pi_{1\text{loop}}(k^2) \quad (2)$$

- (b) Calculate the $\Pi(k^2)$ due to the one-loop diagrams (1), determine the δ_3 counterterm (at the $O(e^2)$ level), and write down the *net* $\Pi(k^2)$ as a function of k^2 .
- (c) Finally, consider the effective coupling $\alpha_{\text{eff}}(k^2)$ of the scalar QED at high momenta. Show that at the one-loop level,

$$\frac{1}{\alpha_{\text{eff}}(k^2)} \approx \frac{1}{\alpha(0)} - \frac{1}{12\pi} \left(\log \frac{-k^2}{m^2} - \frac{8}{3} \right). \quad (3)$$