1. First, a reading assignment: §6.1 of the *Peskin & Schroeder* textbook. Read carefully about *bremmsstrahlung* by scattered electrons in classical electrodynamics and in QED, and pay particular attention to the infrared divergences of cross-sections for emitting soft photons with $\omega_\gamma \to 0$.

   Also, skim through §6.5 about multiple soft photons, real or virtual; never mind the techniques discussed in this section, but the results are important.

2. Now consider the muon's anomalous magnetic moment $a_\mu = \frac{1}{2}(g_\mu - 2)$. Experimentally, it has been measured with a very high precision (up to a fraction of one billionth) and the theoretical calculations have a similarly high precision. At present, there is a very small discrepancy

$$a_\mu^{\rm exp} \; - \; a_\mu^{\rm theory} \;\approx\; (27 \pm 8) \cdot 10^{-10}, \tag{1}$$

   which could be due to inaccurate modeling of the photon-hadron coupling (which affects the theoretical a_μ at the two-loop level), but may also stem from some new particles beyond the Standard Model. (For example, the superpartners, or an extra Higgs scalar, or an axion, or . . . ; loops involving any such particles can affect the muon-muon-photon vertex and hence the muon's anomalous magnetic moment.)

   In this exercise, we consider the effect on the a_μ of just one extra particle field the Standard Model, namely a heavy neutral scalar field S of mass $M \gtrsim 500$ GeV with a small Yukawa coupling g to the muon field Ψ,

$$\mathcal{L} \;\supset\; gS \times \overline{\Psi}\Psi. \tag{2}$$

   Your task is to calculate the effect $\Delta_S a_\mu$ of this scalar field on the muon's anomalous magnetic moment. Then you should use your result and eq. (1) to derive an upper limit on the Yukawa coupling g.

3. Finally, consider the δ^2 counterterm of QED. Calculate both the infinite and the finite parts of this counterterm at the one-loop level, then compare it to the δ_1 counterterm we have calculated in class — *cf.* eq. (83) of my notes (page 18). Verify that $\delta^2 = \delta^1$, including the finite parts of both counterterms.

   The counterterms depend on the regulators (both UV and IR) and on the gauge used for the photon propagators, so use the same gauge and regulators we have used in class: $D = 4 - 2\epsilon < 4$ dimensions to regulate the UV divergence, a tiny photon mass $m_\gamma^2 \ll m_e^2$ to regulate the IR divergence, and the Feynman gauge for the photon propagators. Start by calculating the $\Sigma^{1\,\text{loop}}(\not{p})$ for the off-shell electron momenta p, then take the derivative $d\Sigma/d\not{p}$, and only then take the momentum on-shell, $\not{p} \to m_e$. Note that $\Sigma(\not{p})$ itself is infrared-finite, but its derivative has an IR singularity when the momentum goes on-shell, and that's why you need the IR regulator.

   Note: You should get $\delta^2 = \delta^1$ *before you take the* $D \to 4$ *limit.* If this does not work, check your calculations for mistakes.