1. First, a largish reading assignment: §9.6–9 of the Jackson's textbook about the spherical waves and their multipole expansion. Specifically:

   §9.6 about the scalar spherical waves, their expansion into multipoles, and their radial dependence in the intermediate zone $r \sim \lambda$.

   §9.7 about the EM spherical waves and their expansion into electric and magnetic multipoles.

   §9.8 (first half) about the energy in spherical EM waves. You may skip the second half of this § about the angular momentum.

   §9.9 about angular distribution of the multipole radiation.

2. Second, to test your understanding of the above reading assignment, write down explicit formulae for the electric and magnetic fields for the following multipoles:

   (a) Electric dipole.

   (b) Magnetic dipole.

   (c) Electric quadrupole.

   (d) Magnetic quadrupole.

   For each multipole, assume a *divergent* spherical wave and spell out the complete radial profiles of the electric and magnetic fields in the intermediate zone $r \sim 1/k$. Also, write down explicit angular profiles of the electric and magnetic fields.

3. Finally, a problem about scattering, specifically scattering of EM waves from a small perfectly conducting sphere of radius $a \ll$ wavelength λ.

   (a) Because of skin effect, a perfect conductor acts as a perfect diamagnetic to an oscillating magnetic field. Use this fact to show that the incident EM wave induces an oscillating magnetic dipole moment in the sphere with amplitude

$$\mathbf{m} \;=\; -2\pi a^3 \mathbf{H}_{\rm inc}\,. \tag{1}$$

(b) Besides the magnetic dipole, the wave also induces an oscillating electric dipole moment

$$\mathbf{p} \;=\; +4\pi a^3 \epsilon_0 \mathbf{E}_{\rm inc}\,. \tag{2}$$

Verify this formula, then show that the electric and the magnetic dipole moments are related to each other as

$$\frac{\mathbf{m}}{c} \;=\; -\frac{1}{2}\,\mathbf{n}_0 \times \mathbf{p} \tag{3}$$

(c) Calculate $\mathbf{f}(\mathbf{n})$ due to combined electric and magnetic dipoles and hence the EM fields $\mathbf{E}_{\rm sc}$ and $\mathbf{H}_{\rm sc}$ of the scattered wave in the far zone.

(d) Derive the polarized partial cross-section for scattering from the conducting sphere. Show that for general polarizations

$$\frac{d\sigma}{d\Omega} \;=\; k^4 a^6 \times \left|\mathbf{e}^* \cdot \mathbf{e}_0 \;-\; \tfrac{1}{2}(\mathbf{n}\times\mathbf{e}^*)\cdot(\mathbf{n}_0\times\mathbf{e}_0)\right|^2, \tag{4}$$

while for specific linear polarizations $\perp$ and $\|$ to the scattering plane,

$$\frac{d\sigma^{\perp}}{d\Omega} \;=\; k^4 a^6 \times (1 - \tfrac{1}{2}\cos\theta)^2, \quad \frac{d\sigma^{\|}}{d\Omega} \;=\; k^4 a^6 \times (\tfrac{1}{2} - \cos\theta)^2. \tag{5}$$

(e) Calculate the un-polarized partial cross-section as a function of scattering angle θ. Note that unlike in the dielectric sphere example explained in class, the scattering off a conducting sphere does not have a forward-backward symmetry $\theta \to \pi - \theta$.
Also, calculate the polarization degree $\Pi(\theta)$ of the scattered EM wave for the un-polarized incident wave.

(f) Finally, calculate the net scattering cross-section.