1. Due Wednesday, February 24 (2021).

First, finish the textbook problem 10.2 — calculate to one-loop order the infinite parts of all the counterterms of the pseudoscalar Yukawa theory.

**Hint:** the infinite part of the four-scalar amplitude  $iV(k_1, \ldots, k_4)$  does not depend on the scalar's momenta, so you may calculate it for any particular  $k_1, \ldots, k_4$  you like, onshell of off-shell. I suggest you take  $k_1 = k_2 = k_3 = k_4 = 0$ , so in any one-loop diagram all the propagators in the loop have the same momentum q — which makes evaluating such a diagram much simpler.

Likewise, the infinite part of the one-scalar-two-fermions amplitude  $\Gamma^5(p', p)$  does not depend on the momenta p, p', or k = p' - p, so you may calculate it for any p and p' you like, on-shell or off shell. Again, letting p = p' = 0 makes for a much simpler calculation of the one-loop diagram(s).

2. Due Tuesday, March 2 (2021).

Next, consider the electric charge renormalization in the scalar QED — the theory of a EM field  $A^{\mu}$  interacting with a charged scalar field  $\Phi$ . At the one-loop level, there are two Feynman diagrams contributing to the 1PI two-photon amplitude, namely



(a) Evaluate the two diagrams using dimensional regularization and verify that the net amplitude has form

$$\Sigma_{1\,\text{loop}}^{\mu\nu}(k) = \left(k^2 g^{\mu\nu} - k^{\mu} k^{\nu}\right) \times \Pi_{1\,\text{loop}}(k^2)$$
(2)

(b) Calculate the  $\Pi^{1 \text{ loop}}(k^2)$  due to two diagrams (1), add the  $\delta_3$  counter-term's con-

tribution, then determine the  $\delta_3^{\text{order }\alpha^1}$  coefficient — including its finite part, — and write down the combined  $\Pi_{\text{order }\alpha^1}^{\text{net}}$  as a function of  $k^2$ .

(c) Consider the effective coupling  $\alpha_{\text{eff}}(k^2)$  of the scalar QED at high off-shell momenta,  $k^2 \gg m^2$ . Show that at the one-loop level,

$$\frac{1}{\alpha_{\rm eff}(k^2)} = \frac{1}{\alpha(0)} - \frac{1}{12\pi} \left( \log \frac{-k^2}{m^2} - \frac{8}{3} \right) + O(\alpha). \tag{3}$$

3. Due Tuesday March 2.

Finally, a big reading assignment: my notes on diagrammatic proof of the Ward– Takahashi identities. Please go *carefully* through the algebra, and make sure you understand the diagrammatic proof of the identities.

Update 2/20, 11:45 PM: I have just finished reorganizing my notes on the Ward–Takahashi identities, which ended in two new sets: notes on WT identities and the current conservation will be used in my lectures on 2/24-26/2021, while notes on diagrammatic pfoof of the WT indentities is your reading assignment. In terms of my old notes — which were linked to the earlier version of this homework — your reading assignment is the first  $20\frac{1}{2}$  pages.