1. Let's start with the muon's anomalous magnetic moment $a_\mu = \frac{1}{2}(g_\mu - 2)$. Experimentally, it has been measured with a very high precision and the theoretical calculations have a similarly high precision; however, there is a very small discrepancy

$$a_\mu^{\rm exp} \;-\; a_\mu^{\rm theory} \;\approx\; (26 \pm 8)\cdot 10^{-10}. \tag{1}$$

This discrepancy could be due to inaccurate modeling of the photon-hadron coupling (which affects the theoretical a_μ at the two-loop level), but it may also stem from some new particles beyond the Standard Model such as the superpartners, or an extra Higgs scalar, or an axion, or . . . : The loop diagrams involving any such particles can affect the muon-muon-photon vertex and hence the muon's anomalous magnetic moment.

In this exercise, we consider the effect on the a_μ of just one extra particle field the Standard Model, namely a heavy neutral scalar field S of mass $M \gtrsim 300$ GeV with a small Yukawa coupling g to the muon field Ψ,

$$\mathcal{L} \;\supset\; gS\times\overline{\Psi}\Psi. \tag{2}$$

Your task is to calculate the one-loop-level effect $\Delta_S a_\mu$ of this scalar field on the muon's anomalous magnetic moment.

(a) First, read *carefully* my notes on QED vertex correction , in particular the electron's anomalous magnetic moment's calculation (pages 2–13). Make sure you understand all the algebraic tricks I used to calculate the numerator $\mathcal{N}^\mu$ and split it into terms contributing to the form factors $F_1(q^2)$ and $F_2(q^2)$: You will need similar tricks in this problem.

(b) Draw a 1-loop diagram involving the Yukawa coupling (2) and contributing to the muon-muon-vertex. Evaluate the diagram and bring it to the form

$$\Delta_S\Gamma^\mu \;=\; 2ig^2 \int d\binom{\rm Feynman}{\rm parameters} \int\limits_{\rm red} \frac{d^4\ell}{(2\pi)^4}\, \frac{\mathcal{N}^\mu}{[\ell^2 - \Delta + i0]^{\rm power}} \tag{3}$$

(c) Simplify the numerator in the context of on-shell muons (but not the photon) and bring it to the form suitable for calculating the scalar's contribution $\Delta_S F_1(q^2)$ and the $\Delta_S F_2(q^2)$ to the muon's form factors.

(d) Evaluate the momentum and the Feynman parameter integrals for the $\Delta_S a_\mu = \Delta_S F_2(q^2 = 0)$.
Hint: while integrating over the Feynman parameters, use $M_S^2 \gg m_{\rm muon}^2$ and approximate the integrand accordingly.

(e) Finally, use eq. (1) to impose an upper limit on the Yukawa coupling g for a scalar of mass $M_S = 300$ GeV.

2. Next, consider the δ^2 counterterm in basic QED. Calculate both the infinite and the finite parts of this counterterm at the one-loop level, then compare it to the δ_1 counterterm we have calculated in class — *cf.* eq. (95–96) of my notes on the dressed QED vertex (pages 20-21). Verify that $\delta^2 = \delta^1$, including the finite parts of both counterterms.

   The counterterms depend on the regulators (both UV and IR) and on the gauge used for the photon propagators, so use the same regulators and same gauge as in my notes: $D = 4 - 2\epsilon < 4$ dimensions to regulate the UV divergence, a tiny photon mass $m_\gamma^2 \ll m_e^2$ to regulate the IR divergence, and the Feynman gauge $\xi = 1$.

   Start by calculating the $\Sigma^e_{1\,\rm loop}(\not{p})$ for the off-shell electron momenta p, then take the derivative $d\Sigma/d\not{p}$, and only then take the momentum on-shell, $\not{p} \to m_e$. Note that $\Sigma(\not{p})$ itself is infrared-finite, but its derivative has an IR singularity when the momentum goes on-shell, and that's why you need the IR regulator.

   Note: You should get $\delta^2 = \delta^1$ *before you take the* $D \to 4$ *limit.* If this does not work, check your calculations for mistakes.

3. Finally, a reading assignment: §6.1 of the *Peskin & Schroeder* textbook. Read carefully about *bremmsstrahlung* by scattered electrons in classical electrodynamics and in QED, and pay particular attention to the infrared divergences of cross-sections for emitting soft photons with $\omega_\gamma \to 0$.

   Also, skim through §6.5 about multiple soft photons, real or virtual; never mind the techniques discussed in this section, but the results are important.