REASONS FOR QFT

Q : *Why do we study Quantum Field Theory?*

A1 : *Because it's there.*

A2 : *Because it works.*

A3 : *Because it's the only game in town.*

Explanations:

BECAUSE IT'S THERE: Electromagnetism. Classical electrodynamics is a field theory, and the best way to quantize it is the quantum field theory called QED (Quantum Electrodynamics).

A bit of history: In 1927 Dirac developed old QED: the quantum EM fields coupled to quantum atoms, molecules, nuclei, *etc.*; this theory explained emission, absorption, and scattering of photons by matter. By 1934 he also came up with the quantum field for the relativistic electrons and positrons, but the perturbation theory for the EM and electron fields broke Lorentz symmetry and suffered from badly divergent integrals. Eventually, in 1947–49 Schwinger, Feynman, and Tomonaga constructed the modern QED: a manifestly relativistic perturbation theory with milder divergences and the renormalization techniques for eliminating those divergences from the final answers.

Modern QED works amazingly well and allows to calculate some quantities with great precision. For example, the gyromagnetic factor for the electron's spin has been calculated to 14–digit accuracy,

$$\mathbf{m} \;=\; \frac{-e}{2m_e c}\bigl(\mathbf{L}+g\mathbf{S}\bigr), \qquad g \;\approx\; 2.002\,319\,304\,363^{\pm 1},$$

in good agreement with the experimentally measured value $g \approx 2.002\,319\,304\,361\,8^{\pm 5}$. Indeed, historically it has been a close race between the theorists and the experimentalists who can come up with a better precision for this gyromagnetic factor, but the theoretical and the experimental results were always in agreement with each other.

Because it works: Besides the electromagnetism, the weak and the strong interactions — indeed, all the known elementary particle interactions except gravity — are explained by a quantum field theory called the *Standard Model.* It includes 24 electron-like fermionic fields for the quarks and the leptons, 12 EM-like vector fields for the force carriers, and a scalar Higgs field for breaking the symmetry between the EM and weak forces. We shall explore this theory later in this class.

Besides the relativistic QFT for the high-energy physics, the non-relativistic QFTs are very useful for understanding many aspects of the condensed matter physics. It's particularly useful for effects which involve distances much longer than the distance between the neighboring atoms, especially the critical phenomena, but also superconductivity, superfluidity, Bose–Einstein condensation, *etc.*, *etc.* For the inherently short distance effects in condensed matter, quantum field theories become inapplicable as such, but some QFT techniques do carry over to the short-distance regime.

Because it's the only game in town *for a relativistic quantum theory.* Indeed, a relativistic quantum theory of a single particle — or any *fixed* number of particles — is inconsistent; as we shall see later in class, it has problems with superluminal propagation. To remedy this problem, one must allow for creation of extra particles or their annihilation, so one needs a quantum theory of an arbitrary — and variable — particle number. If we restrict all such particles to be of the same species, or of any finite number of species, then the resulting quantum theory is equivalent to a quantum field theory.

Note: Although classical Einstein gravity is a classical field theory, the quantum gravity is **not** a QFT of any kind. Alas, no QFT can explain the entropies of black hole's horizons: the Bekenshtein–Hawking entropy scales with the black hole's Schwarzschild radius as $S \propto R^2$ while QFT predicts $S \propto R^{3/2}$. Consequently, the quantum micro-physics of BH horizons must go beyond QFT to a theory with infinite number of particle species. Currently, our best candidate theory is the string theory.

Although the string theory involves an infinite number of particle species, almost all of them are superheavy — $M_{\text{heavy}} \gtrsim 10^{18}\,M_{\text{proton}}$, so they are never created in present-day high-energy experiments — and only a finite number of species are light enough to be of phenomenological concern. Moreover, at light particle energies $E \ll M_{\text{heavy}}$, the interactions between the light particles can be described by an effective quantum field theory. And that's why QFT shall rule the particle phenomenology until our accelerators become 10^{15} times more powerful than the LHC.

Field–Particle Duality and other QFT Dualities

Our brains are classical and we have trouble understanding quantum system as such. That is, once we understand what a quantum system describes, we can calculate all kinds of interesting things; but to understand the nature of the system in the first place we need to take a classical (or semiclassical) limit. Some quantum theories have two (or more) very different classical limits, and these two (or more) limits act as *dual descriptions* of the same quantum system.

For example, consider light: Is it a stream of photons or an electromagnetic wave? Turns out, these are two classical limits of exactly same quantum theory. That is, we may start with classical free electric and magnetic field, quantize them, and get a quantum field theory. But when we look for the Hamiltonian eigenstates of that quantum theory, we find arbitrary numbers of identical bosons, each boson being a massless relativistic particle with two transverse polarization states — a photon. On the other hand, we may build a quantum theory of arbitrary number of photons — and mind the Bose statistics. But then in the Hilbert space of that theory we find coherent states which behave exactly like the classical EM fields, and even operators which act exactly like the quantum EM fields. Thus, the quantum theory of photons and the quantum theory of EM fields are exactly the same — the same Hilbert space and the same Hamiltonian.

The same field-particle duality applies to other kinds of fields and particles: some were first discovered as fields — like the EM fields — while other as particles — like the electrons — but the quantum theory always contains both the fields and the particles, and we may take whichever classical limit is more convenient for the problem at hand. The same duality works for the non-relativistic particles and fields in the condensed matter setting, and it's often very useful. For example, the superfluidity of liquid helium can be described by the Landau–Ginzburg field theory, which is the classical limit of the quantum field theory whose quanta are helium atoms. Similar Landau–Ginzburg descriptions work for the Bose–Einstein condensates of cold heavy atoms, and even for the Cooper pairs in superconductors. On the other hand, the quantized sound waves in crystals are often described in terms of quasiparticles — the phonons; similarly, other kinds of waves in condensed matter are also described in terms of quasiparticles.

For other types of dualities, different classical or semiclassical limits of the theory exist for different values of some parameter of the theory. For example, some generalizations of QED contain both electrically charged and magnetically charged particles (magnetic monopoles). However, for small values of $\alpha = e^2/\hbar c$ (in real life, $\alpha \approx 1/137$), the electric charges are approximately pointlike (like the electrons) while the magnetic charges are composite particles made from many quanta, and their radii are much larger than their Compton wavelength. But if we analytically continue the theory to large $\alpha \gg 1$, the magnetic monopoles become pointlike while the electrically charged particles swell into composite clouds of virtual photons and particle-antiparticle pairs. The best way to understand this regime is via the electric-magnetic duality which swaps electric and magnetic fields & charges with each other and maps $\alpha \gg 1$ to $\alpha' = (1/\alpha)$, so the dual theory looks pretty much like the usual QED with $\alpha' \ll 1$.

For the more complicated generalizations of QED — like the non-abelian gauge theories — the electric the magnetic limits of the same quantum can be quite different from each other, but this goes beyond the scope of this class. (But I shall teach the non-abelian gauge theories as such, at the semiclassical level this Fall, and some perturbation theory in the Spring. But I won't get to the E $\leftrightarrow$ M dualities in such theories.)

Finally, there are more complicated kinds of dualities first discovered in the string theory context. For example, the holographic dualities between some gauge theories and supergravity theories in curved spaces with extra dimensions. Hopefully, somebody will teach those dualities in a special-topics class in the next couple of years, but I would not get to them in this class. However, there is a general rule for all such dualities: The quantum theory is exactly the same, it's the (semi)classical limits which look different but are dual to each other.