Unitary matrix: preserves norm
of a complex vector ξ

$$\| U\xi \|^2 = \| \xi \|^2$$

$$(U\xi)^\dagger (U\xi) = \xi^\dagger \xi$$

$$\|$$

$$\xi^\dagger U^\dagger U \xi$$

True $\forall \xi \Rightarrow \boxed{U^\dagger U = 1}$

$$\Rightarrow (U\xi)^\dagger (U\eta) = \xi^\dagger \eta \quad \forall \xi, \eta$$

$$U^\dagger U = 1 \rightarrow U \text{ is invertible}$$

$$\boxed{U^{-1} = U^\dagger}$$

A unitary operator $\hat{U}$ is