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Harm. oscillator

eigenstates $|n,\alpha\rangle$, $E_n = \hbar\omega(n+\frac{1}{2})$

same set of α @ each n.

start w/ ~~the~~ a ground state $|0,\alpha\rangle$

$$\hat{a}^{+}|0,\alpha\rangle = |1, \text{same } \alpha\rangle$$

$$\hat{a}^{+2}|0,\alpha\rangle = \hat{a}^{+}|1,\alpha\rangle = \sqrt{2}\,|2,\alpha\rangle$$

$$(\hat{a}^{+})^{3}|0,\alpha\rangle = \hat{a}^{+}\sqrt{2}\,|2,\alpha\rangle = \sqrt{2}\sqrt{3}\,|3,\alpha\rangle$$

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$$(\hat{a}^{+})^{n}|n,\alpha\rangle = \sqrt{n!}\,|n,\alpha\rangle.$$

$\hookrightarrow$ any $|n,\alpha\rangle$ obtains from $|0,\alpha\rangle$ by acting with $\hat{a}^{+}$ n times.

Suppose no degeneracy. $\rightarrow |n\rangle$, non-deg.

$$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$\langle m|\hat{a}|n\rangle = \sqrt{n}\,\delta_{m,n-1}$$

$$\hat{a}^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$\langle m|\hat{a}^{+}|n\rangle = \sqrt{n+1}\,\delta_{m,n+1} = \sqrt{m}\,\delta_{n,m-1}$$

$$\hat{q} = \sqrt{\frac{\hbar}{2m\omega}}\,(\hat{a} + \hat{a}^{+})$$

$$\hat{p} = \frac{1}{\sqrt{2\hbar m\omega}}\,(-i\hat{a} + i\hat{a}^{+}).$$

$$\left.\begin{array}{l} \rightarrow \langle m|\hat{q}|n\rangle = 0 \\ \langle m|\hat{p}|n\rangle = 0 \end{array}\right\} \text{ unless } m = n \pm 1.$$

In any new state $|\Psi\rangle$

$\langle\Psi|\hat{q}|\Psi\rangle(t)$, $\langle\Psi|\hat{p}|\Psi\rangle(t)$

oscillate harmonically

with frequency ω.

Let $|\Psi\rangle(t=0) = \sum_n c_n |n\rangle$

$$\rightarrow |\Psi\rangle(t) = \sum_n c_n |n\rangle e^{-i\omega t(n+\frac{1}{2})}.$$

$$\langle\Psi|\hat{q}|\Psi\rangle(t) = \sum_{n,m} c_n c_m^{*} \langle m|\hat{q}|n\rangle \times$$
$$\times e^{i\omega(m-n)t}.$$

but $\langle m|\hat{q}|n\rangle = 0$ unless $m = n \pm 1$.

$$\rightarrow \langle\Psi|\hat{q}|\Psi\rangle = e^{+i\omega t} \sum_n c_n c_{n+1}^{*} \langle n+1|\hat{q}|n\rangle$$

+ complex conjugate.

$\rightarrow$ no frequencies except ω

$|n_1, \dots n_N\rangle$

wavefunctions $\psi(q_1, \dots, q_N)$

funct. of ~~depends~~ on all $q_1 \dots q_N$
but not of $p_1 \dots p_N$

state $|n_1, \dots n_N\rangle$

has $\psi(q_1, \dots, q_N) = \psi_{n_1}(q_1)\, \psi_{n_2}(q_2) \times \dots \times \psi_{n_N}(q_N)$

Phonons

$$\hat{\mathcal{H}}_{tot} = \bigoplus_{N=0}^{\infty} \hat{\mathcal{H}}_N$$

$\hat{\mathcal{H}}_N$: Hilbert space of N phonons, ~~other~~ N identical bosons

$\hat{\mathcal{H}}_{tot}$: Hilbert space of the vibrating string

→ Hilbert space of any number of phonons.

∀ state of finite energy, N is finite