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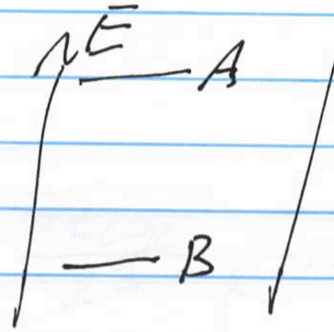
EM fields + Atom(s)

$$\hat{H} = \hat{H}_{EM} + \hat{H}_{atom} + \hat{H}_{int}.$$

$\hat{H}_{int}$ causes transition:

emission & absorption of light

Atomic levels



emission

$|A\rangle$ + photons

$\downarrow$

$|B\rangle$ + 1 more photon

absorption

$|B\rangle$ + photons $\rightarrow$ $|A\rangle$ + 1 less photon.

Fermi Golden rule

transition rate $\Gamma = \dfrac{d(\text{probability})}{d(\text{time})}$.

$$\Gamma = \frac{2\pi}{\hbar} \left| \langle f_m | \hat{H}_{int} | i_n \rangle \right|^2 \times \rho$$

ρ = density of final states

$$= \frac{d\ \#\,\text{final states}}{d\ E_{final}}$$

For atom size $\ll$ wavelength

$$\tilde{H}_{int} = -\vec{d} \cdot \hat{\vec{E}}(\vec{x}_{atom})$$

$\vec{d}$: atom's electric dipole.

$$\langle f_m | \hat{H}_{int} | i_n \rangle = \langle \cancel{f_{atom}}$$

$$= -\langle f_m | \vec{d} | i_n \rangle_{atom} \cdot \langle EM | \hat{\vec{E}} | EM \rangle$$

$$\hat{\vec{E}} = \sum_{k\lambda} \sqrt{\frac{2\pi\hbar\omega_k}{L^3}} \begin{pmatrix} -i \vec{e}_{k\lambda} e^{ikx} \hat{a}_{k\lambda} \\ + i \vec{e}^{\,*}_{k\lambda} e^{-ikx} \hat{a}^{+}_{k\lambda} \end{pmatrix}$$

absorption: $\langle \{n\} - 1_{k\lambda} | \hat{\vec{E}} | \{n\} \rangle$

$$= \cancel{\alpha} \sqrt{\frac{2\pi\hbar\omega_k}{L^3}} \left(-i e^{ikx} \vec{e}_{k\lambda}\right) \cdot \underline{\underline{\sqrt{n_{k\lambda}}}}$$

emission $\langle \{n_{k\lambda}\} + 1_{k\lambda} | \hat{\vec{E}} | \{n\} \rangle =$

$$= \sqrt{\frac{2\pi\hbar\omega_k}{L^3}} \left(i e^{-ikx} \vec{e}_{k\lambda}\right) \cdot \underline{\underline{\sqrt{n_{k\lambda}+1}}}$$

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atom: $\langle B | \vec{d} | A \rangle = \langle A | \vec{d} | B \rangle^{*}$

$\Gamma_{emission} = \Gamma_0 \times \cancel{(n_{k\lambda}+1)} (n_{k\lambda} + 1)$

$\Gamma_{absorption} = \Gamma_0 \times n_{k\lambda}$

same Γ_0

$\Gamma_{absorption} = \Gamma_0 \times n_{\omega}$

$\Gamma_{emission} = \Gamma_0 \times (n_{\omega} + 1)$

$= \Gamma_0 + \Gamma_0 \times n_{\omega}$

spontaneous emission

stimulated emission.

Einstein (1917)

Einstein's argument:

detailed balance in thermal equilibrium

photons & atoms in a cavity.

net transition rates betw atoms

in ~~exis~~ excited state A

and less excited state B

should be equal in both directions

$\# Atoms(A) \times \Gamma_{emission} =$

$= \# Atoms(B) \times \Gamma_{absorption}$

$$\frac{\Gamma_{emission}}{\Gamma_{absorption}} = \frac{\# Atoms(B)}{\# Atoms(A)} = e^{(E_A - E_B)/k_B T}$$

$$= e^{\hbar\omega/k_B T}.$$

$$\frac{\Gamma_{em}}{\Gamma_{abs}} = e^{\hbar\omega/k_BT}$$

by Planck law $\langle n_\gamma \rangle = \frac{1}{e^{\hbar\omega/k_BT} - 1}$

$$\Rightarrow \frac{\langle n_\gamma \rangle + 1}{\langle n_\gamma \rangle} = e^{\hbar\omega/k_BT}$$

$$\boxed{\frac{\Gamma_{emission}}{\Gamma_{absorption}} = \frac{\langle n_\gamma \rangle + 1}{\langle n_\gamma \rangle}}$$

$$\boxed{\begin{aligned} \Gamma_{abs} &= \langle n_\gamma \rangle \cdot \Gamma_0 \rightarrow n_\gamma \Gamma_0 \\ \Downarrow \\ \Gamma_{emission} &= (\langle n_\gamma \rangle + 1)\Gamma_0 \rightarrow (n_\gamma + 1)\Gamma_0 \end{aligned}}$$

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non-equilibrium : # Atom (A) > # atoms (B)

→ population inversion

→ stimulated emission can amplify light.

A particle in 1 dimension

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}).$$

Schrödinger eq-n $\hat{H}|\psi\rangle = E|\psi\rangle$

$$\Leftrightarrow -\frac{\hbar^2}{2m}\psi''(x) + V(x)\psi(x) = E\psi(x).$$

$$\frac{\psi''(x)}{\psi(x)} = \frac{2m}{\hbar^2}(V(x) - E).$$

$\Rightarrow$ 2nd order diff. eq-n

$\rightarrow$ 2 independent solutions.

valid or invalid quantum states $|\psi\rangle$

depending on asymptotic

behavior @ $x \to \pm\infty$.

$x \to +\infty$

if $V(+\infty) > E$

asymptotically $\psi(x) = Ae^{\varkappa x} + Be^{-\varkappa x}$

$$\varkappa = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$e^{-\varkappa x}$ is good. $\rightarrow$ normalizable $\psi(x)$.

$e^{+\varkappa x}$ is bad: it blows up @ ∞.

$\rightarrow$ unnormalizable $\psi(x)$

but its not even asymptotic

General rule: solutions which blow up exponentially (or faster) @ $x \to \pm\infty$ are no ~~got~~ good.

if $V(+\infty) < E$

Asymptotic $\psi(x) = Ae^{ikx} + Be^{-ikx}$

$\rightarrow$ 2 independent unnormalizable solutions, both are ok.

ditto $x \to -\infty$

@ (1) $E < \min(V(+\infty), V(-\infty))$

(2) E between $V(+\infty)$ & $V(-\infty)$

(3) $E > \max(V(+\infty), V(-\infty))$

(1): Each end $x \to \pm\infty$ selects a particular lin combination of 2 solutions $\psi_1(x)$ & $\psi_2(x)$ of the schrödenger eq-n.

for general E, the 2 ends select different combination of ψ_1 & ψ_2
$\rightarrow$ no good solutions.

for some descrete E, both ends select same solution.

(1) For $E < \min(V(+\infty), V(-\infty))$

the energy spectrum is discrete & non-degenerate.

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(2) $V(-\infty) < E < V(+\infty)$

"Boundary" condition @ $x \to -\infty$ select one good solution.

(1 particular combo of ψ_1, ψ_2)

OOH, both solutions are ok @ $x \to +\infty$

$\to \exists$ 1 good solution $\forall E$.

$\Rightarrow$ continuous, non-degenerate spectrum.

(2') $V(-\infty) > E > V(+\infty)$

$\to$ same story.

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(3) $E >$ both $V(+\infty), V(-\infty)$

$\to$ either solution ψ_1 or ψ_2 is ok @ both ends.

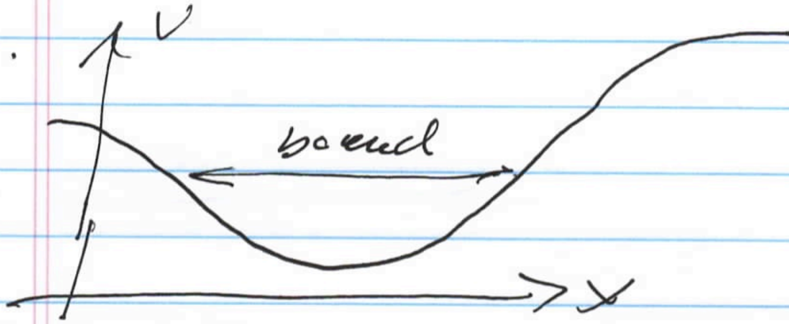
$\to$ continuous, doubly degenerate spectrum.

Discrete Spectrum

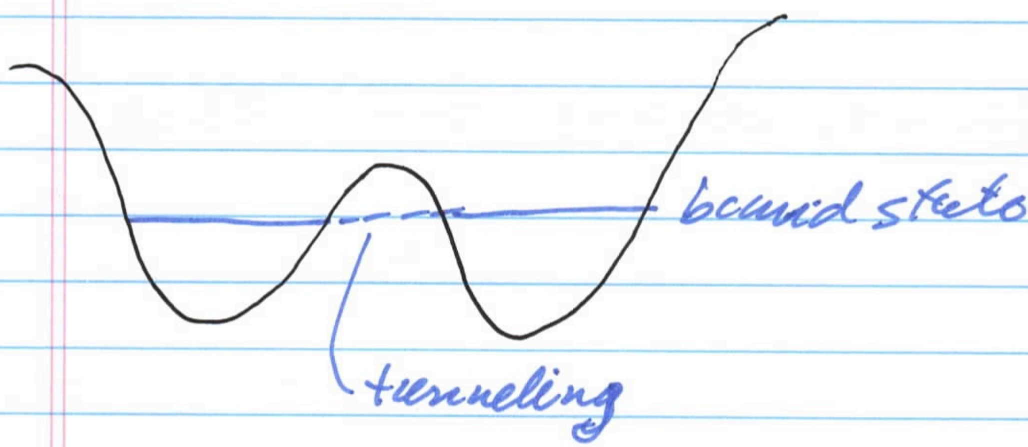
→ bound states

Continuous spectrum

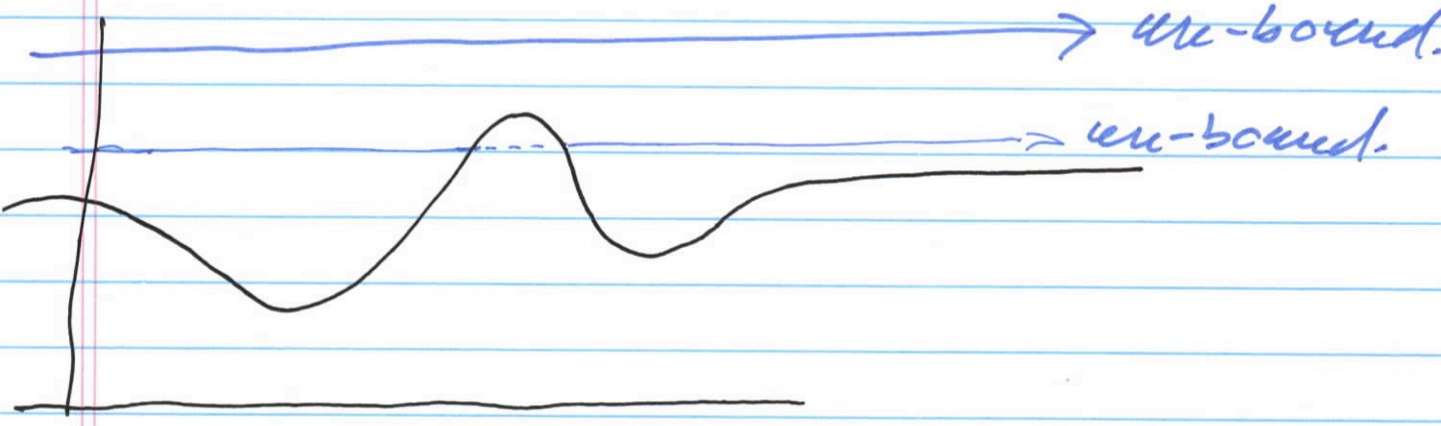
→ un bound states

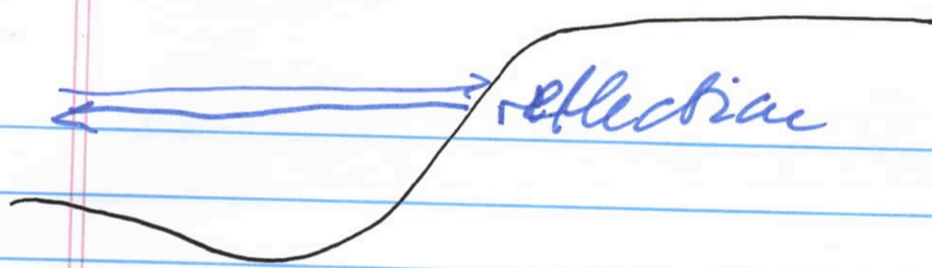


QM: ψ decreases exponentially beyond the classical bounds.

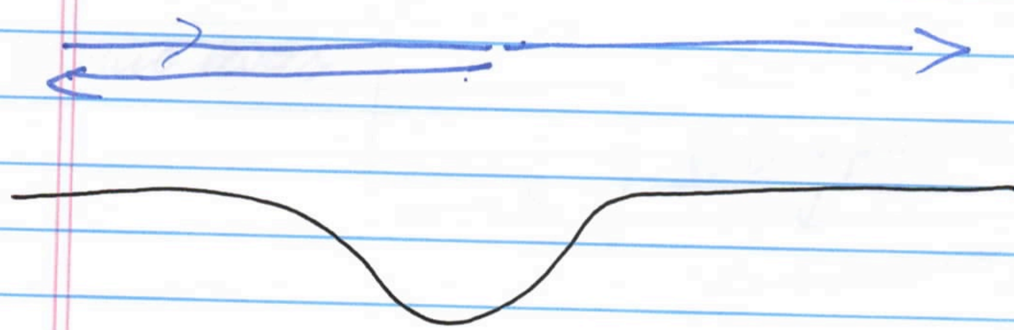


un bound





motion unbound @ left end.



both transmission & reflection