1. First, finish the textbook problem **10.2** — calculate to one-loop order the infinite parts of all the counterterms of the pseudoscalar Yukawa theory.

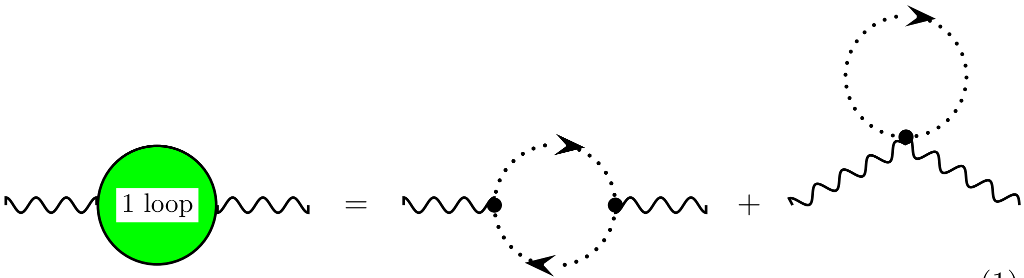
   Hint: the infinite part of the four-scalar amplitude $iV(k_1, \ldots, k_4)$ does not depend on the scalar's momenta, so you may calculate it for any particular $k_1, \ldots, k_4$ you like, on-shell of off-shell. I suggest you take $k_1 = k_2 = k_3 = k_4 = 0$, so in any one-loop diagram all the propagators in the loop have the same momentum q — which makes evaluating such a diagram much simpler.

   Likewise, the infinite part of the one-scalar-two-fermions amplitude $\Gamma^5(p', p)$ does not depend on the momenta p, p', or $k = p' - p$, so you may calculate it for any p and p' you like, on-shell or off shell. Again, letting $p = p' = 0$ makes for a much simpler calculation of the one-loop diagram(s).

   PS: Note that in the $\lambda_{\text{ph}} \to 0$ (but $g_{\text{ph}} \neq 0$) limit, the δ_λ counterterm does not vanish, so the bare Lagrangian has a non-zero 4-pseudoscalar coupling $\lambda_{\text{bare}} \neq 0$. On the other hand, in the $g_{\text{ph}} \to 0$ (but $\lambda_{\text{ph}} \neq 0$) limit, the δ_g counterterm — and hence the bare Yukawa coupling g_{bare} — do vanish along with the g_{ph}. This is an example of a general rule: *barring fine tuning of the coupling parameters, a renormalizable quantum field theory has all the renormalizable couplings consistent with the theorys symmetries*. Hence, *when some physical coupling happens to vanish, the corresponding bare coupling would also vanish only if in is absence the theory would have some extra symmetry.* For example, for $g = 0$ the Yukawa theory gets an extra symmetry $\Phi \to -\Phi$, $\Psi \to \Psi$, so for $g_{\text{ph}} \to 0$ we also have $\delta_g \to 0$ and hence $g_{\text{bare}} \to 0$. On the other hand, there are no extra symmetries for $\lambda = 0$ (but $g \neq 0$), so taking $\lambda_{\text{ph}} \to 0$ would be a fine-tuning while δ_λ and hence λ_{bare} would not vanish along with the physical coupling.

2. Next, consider the electric charge renormalization in the scalar QED — the theory of a EM field A^μ interacting with a charged scalar field Φ. At the one-loop level, there are two Feynman diagrams contributing to the 1PI two-photon amplitude, namely

$i\Sigma^{\mu\nu}_{1\,\text{loop}} =$



(1)

   (a) Evaluate the two diagrams using dimensional regularization and verify that the net amplitude has form

$$\Sigma^{\mu\nu}_{1\,\text{loop}}(k) \;=\; \left(k^2 g^{\mu\nu} \,-\, k^\mu k^\nu\right) \times \Pi_{1\,\text{loop}}(k^2) \tag{2}$$

   Note: the individual diagrams' amplitudes do not have this form. You need to add them up before the 'bad' terms cancel out.

   (b) Calculate the $\Pi^{1\,\text{loop}}(k^2)$ due to two diagrams (1), add the δ_3 counter-term's contribution, then determine the $\delta_3^{\text{order}\,\alpha^1}$ coefficient — including its finite part, — and write down the *combined* $\Pi^{\text{net}}_{\text{order}\,\alpha^1}$ as a function of k^2.

   (c) Consider the effective coupling $\alpha_{\text{eff}}(k^2)$ of the scalar QED at high off-shell momenta, $k^2 \gg m^2$. Show that at the one-loop level,

$$\frac{1}{\alpha_{\text{eff}}(k^2)} \;=\; \frac{1}{\alpha(0)} \;-\; \frac{1}{12\pi}\left(\log\frac{-k^2}{m^2} \,-\, \frac{8}{3}\right) \;+\; O(\alpha). \tag{3}$$

3. Finally, a big reading assignment: My notes on the diagrammatic proof of Ward–Takahashi identities. I shall explain this subject in class on Friday 2/10, but I might skip over some technical details. So your task is to *carefully* go through the algebra, and make sure you understand what's going on.