1. Consider scattering of EM waves from a perfectly conducting sphere of small radius $a \ll$ wavelength λ.

   (a) Because of skin effect, a perfect conductor acts as a perfect diamagnetic to an oscillating magnetic field. Use this fact to show that the incident EM wave induces an oscillating magnetic dipole moment in the sphere with amplitude

   $$\mathbf{m} \;=\; -2\pi a^3 \mathbf{H}_{\rm inc}\,. \tag{1}$$

   (b) Besides the magnetic dipole, the wave also induces an oscillating electric dipole moment

   $$\mathbf{p} \;=\; +4\pi a^3 \epsilon_0 \mathbf{E}_{\rm inc}\,. \tag{2}$$

   Verify this formula, then show that the electric and the magnetic dipole moments are related to each other as

   $$\frac{\mathbf{m}}{c} \;=\; -\frac{1}{2}\,\mathbf{n}_0 \times \mathbf{p} \tag{3}$$

   (c) Calculate $\mathbf{f}(\mathbf{n})$ due to combined electric and magnetic dipoles and hence the EM fields $\mathbf{E}_{\rm sc}$ and $\mathbf{H}_{\rm sc}$ of the scattered wave in the far zone.

   (d) Derive the polarized partial cross-section for scattering from the conducting sphere. Show that for general polarizations of the incident and the scattered waves

   $$\frac{d\sigma(\mathbf{n}_0,\mathbf{e}_0 \to \mathbf{n},\mathbf{E})}{d\Omega} \;=\; k^4 a^6 \times \Bigl|\mathbf{e}^* \cdot \mathbf{e}_0 \,-\, \tfrac{1}{2}(\mathbf{n}\times\mathbf{e}^*)\cdot(\mathbf{n}_0\times\mathbf{e}_0)\Bigr|^2. \tag{4}$$

   In particular, for the linear polarizations $\perp$ and $\|$ to the scattering plane,

   $$\begin{aligned}
   \frac{d\sigma(\perp\to\perp)}{d\Omega} &= k^4 a^6 \times \bigl(1 \,-\, \tfrac{1}{2}\cos\theta\bigr)^2,\\
   \frac{d\sigma(\perp\to\|)}{d\Omega} &= 0,\\
   \frac{d\sigma(\|\to\perp)}{d\Omega} &= 0,\\
   \frac{d\sigma(\|\to\|)}{d\Omega} &= k^4 a^6 \times \bigl(\tfrac{1}{2} \,-\, \cos\theta\bigr)^2.
   \end{aligned} \tag{5}$$

(e) Calculate the un-polarized partial cross-section as a function of scattering angle θ. Note that unlike in the dielectric sphere example explained in class, the scattering off a conducting sphere does not have a forward-backward symmetry $\theta \to \pi - \theta$.
Also, calculate the polarization degree $\Pi(\theta)$ of the scattered EM wave for the un-polarized incident wave.

(f) Finally, calculate the net scattering cross-section and the forward-backward assymetry

$$A = \frac{\sigma(\theta < 90^\circ) - \sigma(\theta > 90^\circ)}{\sigma(\theta < 90^\circ) + \sigma(\theta > 90^\circ)}. \tag{6}$$

2. The rest of this homework is a largish reading assignment. To save time, my lectures on partial wave analysis and on diffration will focus on the scalar waves, while the vector aspects of EM partial waves and EM diffraction are left to self-study in this homework.

(a) Read §10.3–4 of the Jackson's textbook about partial waves for the EM waves. Also, read §10.11 about the optical theorem for the EM waves.

(b) Read §10.6–7 of the Jackson's textbook about vector aspects of diffraction of the EM wave diffraction.