[illegible]

1/A

$$\vec{F} = \vec{\nabla}(\vec{m}\cdot\vec{B})$$

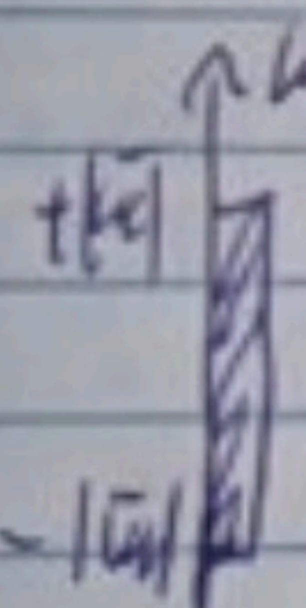
$\vec{B}$: mostly z direction, $\frac{dB_z}{dz} > 0$

$\rightarrow F_z$ measures m_z

$\downarrow$ vert. position on the screen

classically $m_z = |\vec{m}| \cos\theta$

$|\vec{m}|$ fixed, θ $\cos\theta$: random



QM

$m_z =$ either $+m_b$ or $-m_b$

no other values

$m_b = \frac{e\hbar}{2m_e c}$ Bohr magneton.

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measure m_x or m_y inst. of m_z

$\rightarrow$ same story

$m_x = +m_b$ or $-m_b$

$m_y = +m_b$ or $-m_b$

etc.

Real wave

$$X(t) = A\cos(\omega t - \varphi_0)$$
$$= Re[Ae^{i\varphi_0} \times e^{-i\omega t}]$$

$C = Ae^{i\varphi_0}$: complex amplitude

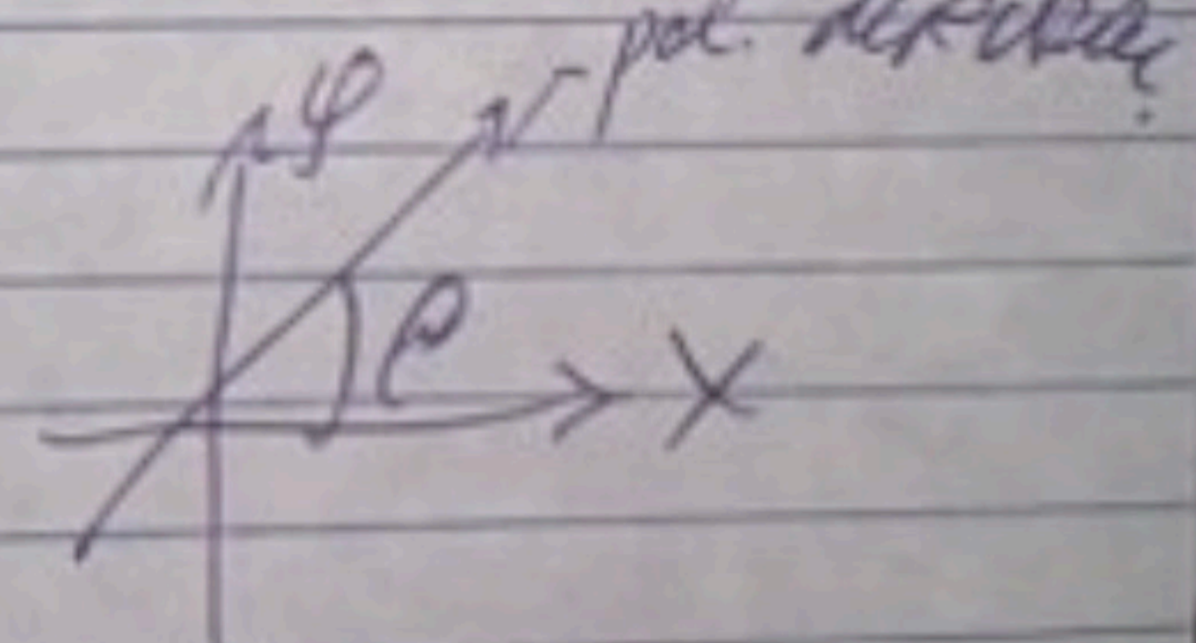
short hand : $X = Ce^{-i\omega t}$, implicit Re.

Plane Light wave, z direction

$$\vec{E}(x,y,z,t) = \vec{\mathcal{E}}\, e^{-i\omega t + ikz}$$

$\vec{\mathcal{E}} = (\mathcal{E}_x, \mathcal{E}_y, 0)$: complex 2d vector. in (x,y) plane.

Linear polarization:



same phase. $\left\{ \begin{array}{l} \mathcal{E}_x = \mathcal{E}_0 \cos\theta \\ \mathcal{E}_y = \mathcal{E}_0 \sin\theta \end{array} \right.$

Circular polarization

$$\left. \begin{array}{l} \mathcal{E}_x = \frac{\mathcal{E}_0}{\sqrt{2}} \\ \mathcal{E}_y = \pm i \frac{\mathcal{E}_0}{\sqrt{2}} \end{array} \right\} \text{same } \mathcal{E}_0$$

$$\vec{E} \sim (\cos\omega t, \pm \sin\omega t, 0)$$

Generic complex $\mathcal{E}_x, \mathcal{E}_y$

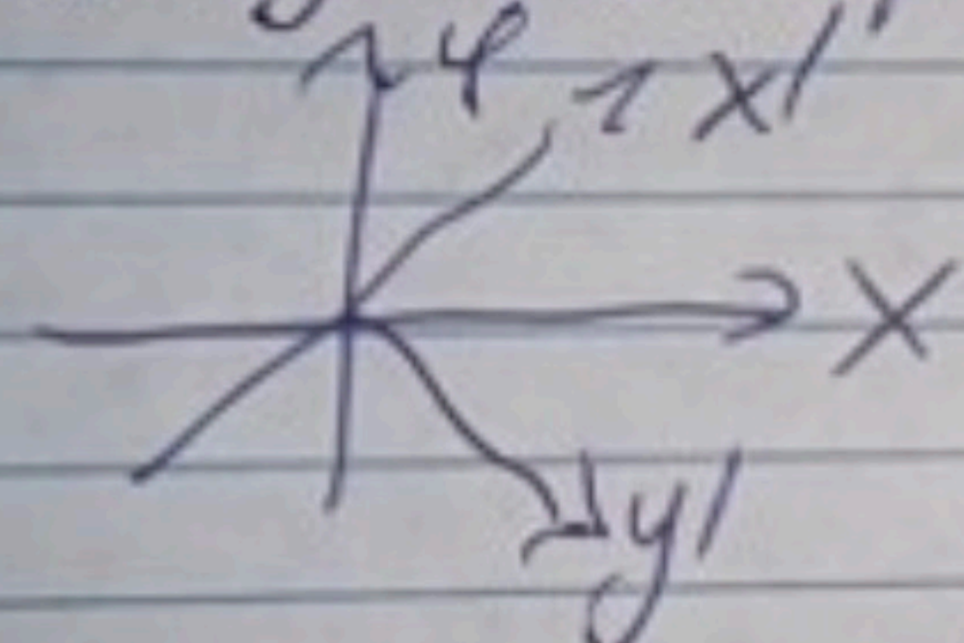
elliptic polarization.

Any polarization $\vec{\mathcal{E}}$ is a lin. comb. of 2 basic polarizations.

but: many diff. bases:

1> x & y linear pol.

2> x' & y' linear pol.



3> L & R circular pol.

etc.

Polarizing filter: projects $\vec{\mathcal{E}}$ onto a specific direction $\hat{n}$

~~[illegible]~~ &

For linear pol. filter

$\mathcal{E}' = \hat{n}(\hat{n}\cdot\vec{\mathcal{E}})$: $\hat{n}$: unit vector in (x,y) plane.

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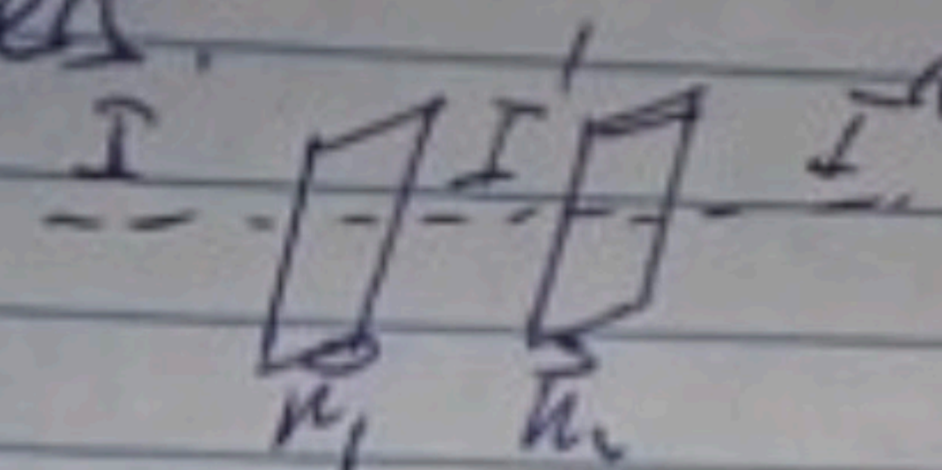
wave power $I \sim |\vec{\mathcal{E}}|^2$

After a filter $I' = |\vec{n}\cdot\vec{\mathcal{E}}|^2 \le I$

2 filters:



$$\frac{I''}{I'_p} = (\vec{n}_1\cdot\vec{n}_2)^2 = \cos(\angle \text{ betw } \vec n_1 \;\&\; \vec n_2)$$

$u_z = +u_0 \Leftrightarrow \vec{n} = (1,0)$ $\hat{x}$

$u_z = -u_0 \Leftrightarrow \vec{n} = (0,1)$ $\hat{y}$

$u_x = +u_0 \Leftrightarrow \vec{n} = (\frac{1}{\sqrt2}, \frac{1}{\sqrt2})$ $+45°$

$u_x = -u_0 \Leftrightarrow \vec{n} = (\frac{1}{\sqrt2}, \frac{-1}{\sqrt2})$ $-45°$

Circular or Elliptic polarizer

$$\vec{\mathcal{E}}' = \vec{n}(\vec{n}^*\cdot\vec{\mathcal{E}})$$

$\vec{n}$: complex unit vector

$$\vec{n}\cdot\vec{n}^* = 1 = |n_x|^2 + |n_y|^2$$

$$\frac{I'}{I} = \frac{|\vec{n}^*\cdot\vec{\mathcal{E}}|^2}{|\vec{\mathcal{E}}|^2}$$

SG: measure $m_y = \pm m_0$

$\updownarrow$

circular pol. filter

$\vec{n} = (\frac{1}{\sqrt{2}}, \pm i\frac{1}{\sqrt{2}})$

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from photons' PoV

$\frac{I'}{I}$ = Probability of a photon goes thru the filter

SG analogy: probability of an atom having the right component of $\vec{m}$.

SG measmt $\Leftrightarrow$ polarization splitter.

2 complementary pol. vectors

n_1 & n_2, $\vec{n}_1^* \cdot \vec{n}_2 = 0$.

$P(\vec{n}_1) + P(\vec{n}_2) = 1$

$\vec{n}$ & $e^{i\varphi}\vec{n}$ are equivalent

$\forall$ phase φ

Quantum states of atom mag. moment $\vec{m}$.

↳ complex 2d vectors.

Q state $|\psi\rangle$: vector in a 2d Hilbert space

Dirac notation: $\langle\psi|$ is a compex conjugate of $|\psi\rangle$

$\langle\psi_1|\psi_2\rangle$ generalizes $\vec{n}_1^* \cdot \vec{n}_2$

any state $|\psi\rangle$ is a lin. comb of 2 basic states

for ex any $|\psi\rangle = \alpha_+ |m_z = +m_0\rangle + \alpha_- |m_z = -m_0\rangle$

for some complex α_+, α_-

any $|\psi\rangle = \beta_+ |m_x = +\rangle + \beta_- |m_x = -\rangle$
for some β_+, β_-

any $|\psi\rangle = \gamma_+ |m_y = +\rangle + \gamma_- |m_y = -\rangle$
for some γ_+, γ_-

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Measuring $m_z = +\hbar/2$

project $|\psi\rangle \to |m_z=+\rangle \cdot \langle m_z=+|\psi\rangle$

analog of $\vec{\varepsilon}\,' = \vec{u}\,(\vec{u}^{\,*}\cdot\vec{\varepsilon})$