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Schrödinger picture of QM.

quantum states evolve w time

$$i\hbar \frac{d}{dt} |\Psi\rangle_s = \hat{H} |\Psi\rangle_s (t).$$

Most operators are time-independent.
→ the way they act is time-indep.

for ex $\hat{p}_i \Psi(\vec{x}, t) = -i\hbar \frac{\partial}{\partial x_i} \Psi(\vec{x}, t)$

same action @ all times.

Exceptions: inherent time dependence

for ex time-dep. external force

$V(x, y, z; \underline{\underline{t}}) \rightarrow \hat{V}(t) = V(\hat{x}, \hat{y}, \hat{z}; t)$

Heisenberg picture:

states $|\Psi\rangle_H$ are time independent

but the operators evolve w time.

Related to the Schrödinger picture

by unitary equivalence

$\hat{W}(t) = \hat{U}^{-1}(t, t_0)$, fixed t_0

usually $t_0 = 0$.

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$$|\psi\rangle_H(t) = \hat{U}^{-1}(t,t_0)\,|\psi\rangle_S(t)$$

$$= \cancel{\hat{U}^{-1}(t,t_0)}\left(\cancel{\hat{U}(t,t_0)}\,|\psi_S\rangle(t_0)\right)$$

$$= |\psi_S\rangle(t_0) \text{ @ all times } t.$$

$\Rightarrow$ same $|\psi_H\rangle$ @ all t.

$$\Rightarrow \hat{A}_H(t) = \hat{U}^{-1}(t,t_0)\,\hat{A}_S(t)\,\hat{U}(t,t_0)$$

$$\frac{\partial \hat{U}}{\partial t} = \frac{1}{i\hbar}\hat{H}(t)\,\hat{U}(t,t_0)$$

$$\frac{\partial \hat{U}^{-1}}{\partial t} = -\hat{U}^{-1}\frac{\partial U}{\partial t}\hat{U}^{-1} = \frac{-1}{i\hbar}\hat{U}^{-1}\hat{H}\cancel{\hat{U}\hat{U}^{-1}}$$

$$= \frac{-1}{i\hbar}\hat{U}^{\dagger}(t,t_0)\,\hat{H}(t).$$

$$\frac{d}{dt}\hat{A}_H(t) = \frac{-1}{i\hbar}\hat{U}^{-1}(t,t_0)\,\hat{H}(t)\,\hat{A}_S(t)\,\hat{U}(t,t_0)$$

$$+ \frac{1}{i\hbar}\hat{U}^{-1}(t,t_0)\,\hat{A}_S(t)\,\hat{H}(t)\,\hat{U}(t,t_0)$$

$$+ \hat{U}^{-1}(t,t_0)\,\frac{\partial A_S}{\partial t}\,\hat{U}(t,t_0)$$

$$= \hat{U}^{-1}(t,t_0)\left(\frac{[A_S, \hat{H}_S]}{i\hbar} + \frac{\partial A_S}{\partial t}\right)\times$$

$$\times\,\hat{U}(t,t_0)$$

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$$\boxed{\frac{d\hat{A}_H}{dt} = \frac{1}{i\hbar}[A_H(t), \hat{H}_H(t)] + \left(\frac{\partial \hat{A}}{\partial t}\right)_H}$$ Heisenberg equation

~~contain~~ takes inherent time-depence

$\frac{\partial \hat{A}_s}{\partial t}$ converted to Heis. picture

In particular $\frac{d}{dt}\hat{H}_H = \cancel{\frac{\partial A}{\partial A}} \left(\frac{\partial \hat{H}}{\partial t}\right)_H$

When there are no time-dep. external forces, $\frac{\partial \hat{H}}{\partial t} = 0$

Then $\frac{d\hat{H}_H}{dt} = 0 \rightarrow \hat{H}_H$ is time-indep.

Moreover $\hat{H}_H = \hat{H}_s$

bec. $\hat{u}^{-1} = \exp(+i(t-t_0)\hat{H}_s/\hbar)$

$\Rightarrow$ commutes with $\hat{H}_s$

$\Rightarrow \hat{H}_H = \hat{u}^{-1}\hat{H}_s\hat{u} = \hat{H}_s$

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The Heisenberg-Dirac eq-n

$$\frac{d}{dt}\langle\Psi|\hat{A}|\Psi\rangle = \langle\Psi|\frac{\partial A}{\partial t}|\Psi\rangle + \frac{1}{i\hbar}\langle\Psi|[\hat{A},\hat{H}]|\Psi\rangle$$

is the same in all pictures.

object	Schrödinger picture	Heisenberg picture
physical states	time-dep.	time-indep.
Most operators	time-indep	time-dep.
	$\updownarrow$	$\updownarrow$
bases of states having def. values of some observable	time-indep.	time-dep

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A conserved observable $\hat{A}$

→ ~~time-independent~~ $\langle \hat{A} \rangle_t = \langle \psi | \hat{A} | \psi \rangle$

$\forall$ initial $|\psi\rangle$.

By H-D eq-n, this requires.

$$\frac{\partial \hat{A}}{\partial t} + \frac{1}{i\hbar}[\hat{A}, \hat{H}] = 0.$$

usually, no inherent time-dep

$$\frac{\partial \hat{A}}{\partial t} = 0 \rightarrow \boxed{\text{need } [\hat{A}, \hat{H}]}$$

$\boxed{\text{conserved observables are compatible with the energy}}$

symmetries are usually assoc. with conserved operators.

⇒ Usually, easier to diagonalize the symmetry generators than $\hat{H}$

⇒ $\hat{H}$ becomes block-diagonal in the symmetry eigenbasis

→ easier to ~~diagonal~~ diagonalize